Ay 126 Homework #1 Solutions

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(6.) According to the RRL designation, the relevant transitions are the following:

C548α ⇒ $n = 549 \rightarrow 548$
C518α $n = 519 \rightarrow 518$

Note that the Rydberg formula applies for transitions involving distant electrons of heavy elements, s.t. the effective nuclear charge is the same as that for hydrogen, since all but one of the nuclear charges have been screened by the other electrons.

Thus, $R_C = \left(1 + \frac{m_e}{12 m_H}\right) R_H$

So C548α : 39.85 MHz
C518α : 47.17 MHz

We cannot detect H, He, O & N RRL lines from the WNM/CNM due to insufficiently high temperatures to result in partial ionization. These chemical species are neutral/molecular in these ISM phases, as opposed to C, which can be partially ionized.

(7.) The major Fraunhoffer lines are the following: (from NIST)

A	O_2	7594 (Å)	–
B	O_2	6867	–
C	Hα	6563	$n=2 \rightarrow n=3$
D_1	Na I	5896	$2p^6(3s^1\ {}^2S_{1/2} \rightarrow 3p^1\ {}^2P_{1/2})$
D_2	Na I	5890	$2p^6(3s^1\ {}^2S_{1/2} \rightarrow 3p^1\ {}^2P_{3/2})$
E_2	Fe I	5270	$3d^7 4s^1\ {}^3F_2 \rightarrow 3d^6 4s^1 4p^1\ {}^3D_1$
F	Hβ	4861	$n=2 \rightarrow n=4$
G	Fe I	4308	$3d^7 4s^1\ {}^3F_3 \rightarrow 3d^7 4p^1\ {}^3G_4$
H	Ca II	3968	$3p^6(4s^1\ {}^2S_{1/2} \rightarrow 4p^1\ {}^2P_{1/2})$
K	Ca II	3934	$3p^6(4s^1\ {}^2S_{1/2} \rightarrow 4p^1\ {}^2P_{3/2})$

We have not indicated the transition for the A & B lines since it is due to terrestrial oxygen. Such telluric absorption is removed from spectra prior to analysis.

A note about spectrosopic terms: (also recall selection rules)

$^{2S+1}\mathcal{L}^{P}_{J}$ $\quad P = \begin{cases} \text{even} & \sum l_i \text{ even} \\ \text{odd} & \sum l_i \text{ odd} \end{cases}$ $\quad l_i$: orbital angular momentum quantum #s of electrons

$\vec{J} = \vec{L} + \vec{S} \Rightarrow |J| = |L+S| \ldots |L-S|$

$0 \le l \le n$
$m_l = -l \ldots l$
$m_s = \pm 1/2$

multiplicity $g = (2S+1)(2L+1)$
consider all possible combinations of m_l & $m_s \Rightarrow S, L$

$\mathcal{L} = S, P, D, F \ldots$ for $L = 0, 1, 2, 3 \ldots$

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(2) If there is no radial velocity difference between Palomar & HST, then the only difference in the observed wavelengths is caused by the index of refraction of Earth's atmosphere.

We have: $n_{vacuum} = 1$, $n_{air} = 1.00029$ at STP.
Let us assume that n_{air} is approximately constant, although in reality it varies spatially & temporally.

Let us first compute the vacuum wavelengths, which correspond to the observed HST wavelengths. We use the Rydberg formula, where $R \approx 1.0973 \times 10^{-3}$ Å,

$$\frac{1}{\lambda_{vac}} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

n_f final principal quantum number
n_i initial principal quantum number

To compute the observed wavelength at Palomar, we use

$$\lambda_{obs} = \frac{\lambda_{vac}}{n_{air}}$$

⇒

Line	λ_{vac} (Å)	λ_{obs} (Å)
Hα	6563.35	6561.45
Hβ	4861.74	4860.33
Hγ	4340.84	4339.58
Lyα	1215.44	–
Lyβ	1025.52	–
Lyγ	972.35	–

The atmosphere's refractive index results in $\Delta\lambda \sim 1-2$ Å

Note that these wavelengths are in the UV, ($\gtrsim 3000$ Å) & thus cannot be transmitted to the ground

(3) First, let us determine the relative intensity of the Hα & Dα lines. For emission,

$$\frac{I(D\alpha)}{I(H\alpha)} = \frac{n_e\, n(D\alpha)\, E_{D\alpha}\, \alpha_{eff,H\alpha}}{n_e\, n(H\alpha)\, E_{H\alpha}\, \alpha_{eff,H\alpha}}$$

where we have assumed that the transition rates are the same for H & D.

$$\Rightarrow \frac{I(D\alpha)}{I(H\alpha)} \sim \frac{n(D\alpha)}{n(H\alpha)} \sim \frac{3\times10^{-5}}{0.75} \sim 4\times10^{-5}$$

Next, let us calculate the isotopic shift of Deuterium relative to Hydrogen. The two main causes of the isotopic shift are the volume effect, & the mass effect, the latter of which dominates for light atoms. An estimate of the shift is,

$$\frac{R_D}{R_H} = \frac{1+\frac{m_e}{m_H}}{1+\frac{m_e}{m_D}} \approx \left(1+\frac{m_e}{m_H}\right)\left(1-\frac{m_e}{m_D}\right) \approx 1+\frac{m_e}{m_D} \quad \text{where } m_D \approx 2m_H.$$

Since the Rydberg constant is proportional to the reduced mass, $R \propto \mu \propto \frac{m_1 m_2}{m_1+m_2} = \frac{\cancel{m_2} m_1}{\cancel{m_2}\left(1+\frac{m_1}{m_2}\right)}$

$$\Rightarrow \frac{1}{\lambda_D} = R_D\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = R_H\left(1+\frac{m_e}{2m_H}\right)\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = \left(1+\frac{m_e}{2m_H}\right)\frac{1}{\lambda_H}$$

$$\lambda_D = \lambda_H\left(1 - \frac{m_e}{2m_H}\right) = (6563\ \text{Å})\left(1 - \frac{1}{2\cdot 1836}\right) \qquad \text{where } \frac{m_e}{m_H} = \frac{1}{1836}$$

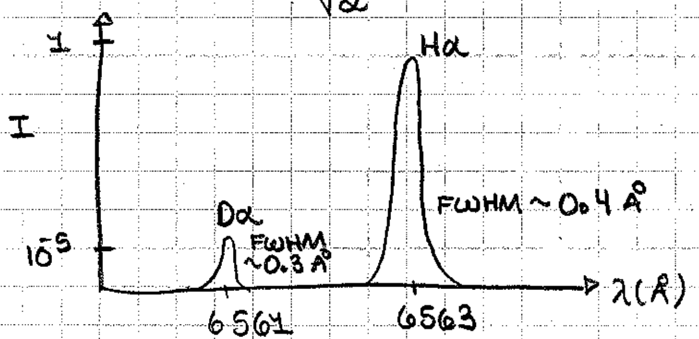
$$\Rightarrow \lambda_D = 6561\ \text{Å}$$

Lastly, we need to calculate the widths of the lines. This is due to thermal Doppler broadening,

$$\Delta\nu = \nu_0\sqrt{8\ln 2\,\frac{k_B T}{m_0 c^2}} \qquad \text{where } T \sim 8000\ \text{K},\ m_0 = m_H \text{ for Hydrogen}$$

$$\Rightarrow \Delta\lambda_H = 0.44\ \text{Å}$$

$$\text{so } \Delta\lambda_D = \frac{\Delta\lambda_H}{\sqrt{2}} = 0.31\ \text{Å}$$



For comparison to Hébard+2001, convert from wavelength to velocity

$$\text{e.g., } \text{FWHM}(\text{km s}^{-1}) = \frac{\text{FWHM}(\text{Å}) \times c\,(\text{km s}^{-1})}{\lambda_0(\text{Å})}$$

(4.) The only difference between positronium & Hydrogen is the reduced mass, i.e. the Rydberg constant, since it is still a hydrogenic atom.

$$\Rightarrow R_P = \left(1 + \frac{m_e}{m_e}\right)^{-1} R_H = \frac{1}{2} R_H \quad \Rightarrow \quad \lambda_P = 2\lambda_H$$

So Hα $\lambda = 1312.6$ nm

Lyβ $\lambda = 205.2$ nm

(5.) The wavelength features observed in the Wolf-Rayet stars are: 4200, 4340, 4541, 4686, 4860, 5411, 6560 Å

From the Bohr model for the atom, we have wavenumbers

$$K = \frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \qquad \text{where } Z \text{ is the atomic number of a given hydrogenic atom.}$$

$\Rightarrow K = 1/\lambda$: $(2.381,\ 2.304,\ 2.202,\ 2.134,\ 2.058,\ 1.848,\ 1.524) \times 10^6$

K/R: 0.217, 0.210, 0.201, 0.195, 0.188, 0.168, 0.139

The transitions λλ6560 & λλ4860 correspond approximately to Hα & Hβ (Z=1). Thus, we know that

$$\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = Z^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = Z^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Assuming $Z = 2$, then $n_f = 4$ & $n_i = 6$ etc.

$n_f = 4$ & $n_i = 8$

This corresponds to a Paschen series ($n_f = 4$) for He II. This makes sense, since the outer H envelope has been ejected to expose to ionized He core.