Ay126 Interstellar Medium: Homework 2

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Due 24 April COB @TA's mailbox

The first three problems are pure pedagogy and are designed to test your understanding of basic atomic spectroscopy and also introduce you to atomic spectroscopy data bases (catalog of energy levels, Grotrian diagrams etc.; see class web page for some pointers). The last two problems are motivated by dramatic developments resulting from far infrared (FIR) missions (particularly Herschel). ALMA, a new facility, has superb sensitivity in the sub-mm bands. These developments allow for the the study of the Universe through fine structure lines (FSL). Separately, rapid improvements in sensitivity and pixel format of FIR detectors are opening up new new paths[1] for FSL based studies.

[1] **Pedagogical exercise in Quantum Defects & Grotrian diagrams.** *For all three problems listed below: Only after you have completed your calculation you are expected to search the literature and cross check your answer. Include references to the data base/papers in your reference(s) in your answer.*

A) The ionization potential of Na I is 5.139 eV. The first member of the principal series is the famous D doublet ($\lambda\lambda 5890, 5896$ Å). Using the classical Rydberg approach (replete with quantum defects) infer the wave numbers for the next four members of the series. Construct the energy level diagram and write down the spectroscopic terms for each level. [5 pts]

B) The ground state transition of CIV is a famous diagnostic of hot gas from quasars as well as the IGM. The ionization potential for CIV is 64.476 eV. The first member of the principal series, with a wave number of $64555.6\,\mathrm{cm}^{-1}$, is well known. Write down the two spectroscopic terms for this transition. Then compute the wave number (and wave length) of the second member of this series. [5 pts]

C)[2] The following absorption lines were observed in the spectrum of a neutral gas: $[1.301, 2.471, 2.900, 3.107] \times 10^4\,\mathrm{cm}^{-1}$. Fit the data to a classical Rydberg model and determine the atom in question![3] [10 pts]

[1]The great progress in FSL became only apparent to me whilst preparing for this class. Following my lecture on FSL I started think of new missions, based on small diameter telescopes but with large field-of-view (aimed at low surface brightness), aimed for a thorough chemical examination of the IGM. In fact, I have already started a conversation with Zmuidzinas who is the world's expert on large format FIR detectors.

[2]This is an interesting problem in that it shows the "finger printing" ability of atomic spectroscopy. A bit tedious, though.

[3]Hint: the ionization potential can be a clue to the atom.

[2] **Ground State of elements with filled and half-filled subshells.**[4] A shell is distinguished by its energy quantum number, n and has $2n^2$ orbitals. A sub-shell is distinguished by the angular momentum l and can accommodate $2(2l+1)$ orbitals. Study the ground state spectroscopic term of the elements in the periodic table. Explain why the ground spectroscopic term of all noble elements and elements with half-filled sub-shells (cf. Nitrogen) is ${}^1S_0$. [5 points]

[3] **Pedagogical Exercise: Spectroscopic Terms.**[5] Derive the spectroscopic terms for Titanium $3d^2$. Present your analysis. [Extra point: you may wish to check out if Hund's rules applies to the resulting terms.] [10 points]

[4] **An Astronomical Survey of Fine Structure Lines.** Survey the literature and list FSL of atoms and ions detected from astrophysical sources: atom/ion, wavelength/wavenumber, spectroscopic terms, telescope and observed object(s). I expect that at least 10 FSL will be reported. [10 pts]

[5] **Fine Structure Separation.** In the class I presented[6] simple formula for the energy levels of FSL (with respect to the energy level of the unperturbed level). Apply this formula to the lines you listed in the previous problem and see how well the simple analytical model fits the data. [5 pts]

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[4] After correctly answering this question you will understand why the spectroscopic term of the ground state of Oxygen and Carbon is the same. However, there is a distinction between the energy ordering of the J levels, cf. Hund's third rule.

[5] You may wish to review Lecture 5 on the class URL.

[6] also can be found in one of the notes in Lecture 5 (class URL)