An additional constraint for FRBs

S. R. Kulkarni

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There is a significant clue provided by the precision of the arrival time, t_a, as a function of frequency, ν. For propagation through cold plasma we expected that $t_a \propto \nu^p$ where $p = -2$. However, this expectation is true only when $2\pi\nu$ is *significantly larger than the plasma frequency*, ω_p. Since the latter is related to the density of electrons, n_e, deviations from the expected -2 can diagnose n_e.

For one of the FRBs, $p = -2 \pm 0.003$ derived from observations in the 1.4-GHz band (central frequency, ν_0). The deviation from -2 are mostly due to measurement errors. However, let us assume that the index p is really, say $p = -2.006$ (twice σ in one direction).

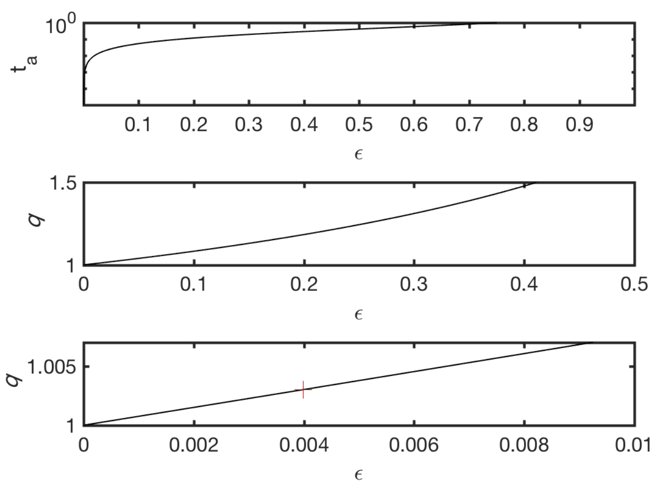

Figure 1: Top: arrival time of a signal assuming $L = 1$ and $c = 1$ (and thus the unit is about 30 ps) as a function of $\epsilon = \omega_p^2/\omega^2$. Middle: Over a small range, $[\epsilon, \epsilon + \delta\epsilon]$, t_a is modeled as a power law and the power index, q as a function of ϵ is obtained by numerical modeling. Bottom: Zoom of the previous panel.

The group velocity of a wave traveling through ionized plasma is

$$v_g = c\left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2} \tag{1}$$

where

$$\begin{aligned}\omega_p = 2\pi\nu_p &= \left(\frac{4\pi n_e e^2}{m_e}\right)^{1/2} \\ \nu_p &= 8.978 n_e^{1/2}\,\mathrm{kHz}.\end{aligned} \qquad (2)$$

The arrival time, as measured with respect to high (infinite) frequency, is

$$t_a(\omega) = \frac{L}{c} - \int_0^L \frac{dl}{v_g} \qquad (3)$$

where l is the coordinate along the line-of-sight to the source and L is the distance to the source. A plot of t_a as a function of $\epsilon = \omega_p^2/\omega^2$ can be found in Figure 1. The arrival time varies with frequency and thus over any given small range of frequency the arrival time curve can be approximated over the frequency range by a power law model, $t_a(\epsilon) \propto \epsilon^{q(\nu)}$. The resulting power law index, $q(\nu)$, is shown in the lower two panels of Figure 1. As expected for $\epsilon = 0$ we do find $q = 1$, which corresponds to $p = -2$.

For the problem at hand we have $p = 2.006$. This value of p translates to $q = +1.003$. From Figure 1 we find that q is obtained for $\epsilon_* = 0.004$ (marked by "+" in the bottom panel of Figure 1). Thus the plasma frequency is

$$\nu_p = \sqrt{\epsilon_* \nu_0^2} = 88.5\,\mathrm{GHz}. \qquad (4)$$

Setting this to the plasma frequency in Equation 2 we find $n_e = 9.7 \times 10^{13}\,\mathrm{cm}^{-3}$ – another constraint on n_e. Note that this constraint is independent of L.