Ay126: Homework 5

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Due COB 7 days @TA's desk, past seven days from the posting date

Pure Hydrogen HII region. This is a pedagogical exercise designed to sharpen your understanding of basic nebular physics. The temperature of an HII region is determined by the head added to the ionized plasma by the kinetic energy of the ejected photoelectrons which is then balanced by radiative decay of metal lines excited by collisions with electrons, recombination radiation (RR)[1] and free-free radiation (FF; $\mathcal{C}$27).[2] Usually, at least, in a typical graduate ISM course such as Ay126, Hydrogen line radiation (e.g. Balmer series) is not included since in the simplified version these lines result from recombined atoms which successively undergo radiative decay (until they reach $n =$ state; assuming case B).

The Trapezium θ^1 Ori, a set of four hot stars, is the primary source of ionizing stars which power the Orion nebula ($\mathcal{C}$28). For this pedagogical problem we will assume that the nebula is composed of *pure Hydrogen*. For simplicity we will assume that the only ionizing star is θ^1 Ori C (see Table 28.1 for stellar parameters) and that the nebula is ionization bounded. For the purpose of the questions below, assume density $n_H = 100\,\mathrm{cm}^{-3}$ and that Case B holds.

1. **Photoelectric heating.** Approximate the spectrum of the ionizing star by a black body. Determine the heatng rate in the two approximations discussed in §27.1. Do you understand why the heating rate is lower in the center of the nebula, relative to that at the edges. Below compute all quantities for both these two heating rates. 5 pts

2. **Cooling.** Compute the recombination radiation (cf. §27.3.1) and free-free emission as a function of temperature (say between 10^4 K and 4×10^4 K; use tabulated entries and interpolate and extrapolate to higher temperatures, as needed). 5 pts

[1]as extensively discussed in the class RR is the kinetic energy lost by electrons when they recombine with protons.

[2]Unless stated otherwise, Chapters ($\mathcal{C}$), Section (§) and Table numbers refer to the ISM book by Draine.

3. **Thermal balance.** Equate the heating and cooling rate to determine the temperature of the ionized plasma in the two heating approximations. 5 pts

4. **Fractional Ionization and Strömgren Radius.** Compute the fractional ionization of the gas and the radius of the Strömgren sphere. Comment on how these two quantities would be different were the nebula would have had cosmic composition. 5 pts

5. **Collisional Excitation of Hydrogen: Hα and Hβ.** In view of the higher nebular temperatures it is worth checking if there is significant cooling due to (electron) collisional excitation of neutral hydrogen. Compute the cooling rate in Hα and Hβ that arises from electron excitation. 10 pts

6. **Collisional Excitation of Hydrogen: Two Photon Emission.** Some collisions will result in excitation to the 2^2S level. Also, the 2^2S level can be populated by radiative decay of an H atom which was to 3^2P level by collisions. The the excited atom can decay by two-photon process or get shifted to the 2^2P level by a collision. Compute the net two-photon cooling rate. 15 pts

7. **Recombination Line and Two-Photon Radiation.** Compute the volumetric emission rate (energy per unit volume per unit time) of Hα, Hβ and two-photon decay *that result from recombinations.* 5 pts

8. **Higher Density.** What would change (in terms of cooling radiation) if $n_H = 10^4\,\mathrm{cm}^{-3}$? (*pro bono* question).

Background material: I recommend Osterbrock & Ferland (Chapters 1–4). You would also benefit from reading Draine ($\mathcal{C}$14 and $\mathcal{C}$27).

References for data: recombination coefficient (§14.1), two photon emission and related processes (§14.2.4), Table 3.16 of Osterbrock & Ferland (collisional excitation of HI), free-free emission (§27.3.2) and Recombination Radiation (§27.3.1).