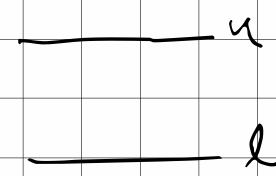
Detailed balance

$$X(l) + Y \longrightarrow X(u) + Y$$



Here $X(u)$, $X(l)$ are the two energy states of X

Assume Y does not change (internal) state.

In equilibrium the upward (excitation) rate should match the downward (de-excitation) rate.

$$n_e n_l \sigma_{lu} = n_e n_u \sigma_{ul}.$$ Use Thermodynamics:

$$\boxed{\therefore \quad \frac{\sigma_{lu}}{\sigma_{ul}} = \frac{n_u}{n_l} = \frac{\tilde{g}_u}{\tilde{g}_l} \exp\left(-\frac{E_{ul}}{kT}\right)}$$

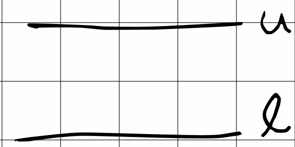
It is traditional to list the de-excitation coffeciant (since it does not have the strong exponential dependence)

If expressed in energy units

$$(E_{ul}+E)\,\sigma_{lu}(E_{ul}+E) = \frac{\tilde{g}_u}{\tilde{g}_l} E\, \sigma_{ul}(E)$$

2-level system:

Chapters 6,



n_l = number density of atoms in "l"ower state

n_u = number density of atoms in "u" state

u_ν = photon number density $\quad \nu, \nu + d\nu$

$$\left(\frac{dn_u}{dt}\right)_{l\to u} = -\left(\frac{dn_l}{dt}\right)_{l\to u} = n_l B_{lu} u_\nu$$

$$\left(\frac{dn_l}{dt}\right)_{u\to l} = -\left(\frac{dn_u}{dt}\right)_{u\to l} = n_u (A_{ul} + B_{ul} u_\nu)$$

$$u_\nu(\text{Black Body}) = \frac{4\pi}{c} B_\nu$$

$$= \frac{4\pi}{c} \cdot \frac{1}{\lambda^2} \cdot 2 \cdot h\nu \frac{1}{e^{h\nu/kT} - 1}$$

$$= \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1}$$

$$B_{ul} = \frac{c^3}{8\pi h\nu^3} A_{ul}$$

$$\frac{B_{lu}}{g_u} = \frac{B_{ul}}{g_l}$$

so only free parameter is A-coefficient

Photon occupation number

$$\bar{n}_\gamma \equiv \frac{c^2}{2h\nu^3} \bar{I}_\nu$$ ← averaging over all directions

$$= \frac{c^3}{8\pi h\nu^3} u_\nu$$

For black-body

$$n_\gamma = \frac{1}{e^{h\nu/kT} - 1}$$

$$\left(\frac{dn_l}{dt}\right)_{u\to l} = n_u A_{ul} (1 + \bar{n}_\gamma)$$

$$\frac{1}{g_u} \left(\frac{dn_u}{dt}\right)_{l\to u} = \frac{1}{g_l} n_l A_{ul} \bar{n}_\gamma$$

We will replace $\langle\sigma v\rangle_{ul} \to k_{10}$

$\langle\sigma v\rangle_{lu} \to k_{01}$

$$\frac{1}{g_1} k_{01} = \frac{1}{g_0} k_{10} \exp - \frac{E_{10}}{kT_{gas}}$$

n_c = number density of collider

$$\frac{dn_1}{dt} = n_0\left[n_c k_{01} + \bar{n}_\gamma \frac{g_1}{g_0} A_{10}\right] - n_1\left[n_c k_{01} + (1+\bar{n}_\gamma) A_{10}\right]$$

In steady-state $\frac{dn_1}{dt} = 0$

$$\frac{n_1}{n_0} = \frac{n_c k_{01} + \bar{n}_\gamma (g_1/g_0) A_{10}}{n_c k_{10} + (1+\bar{n}_\gamma) A_{10}}$$

Limits:

In the limit of no background photons

$\bar{n}_\gamma \to 0$

$$\frac{n_1}{n_0} = \frac{n_c k_{01}}{n_c k_{10} + A_{10}}$$

$$= \frac{k_{01}}{k_{10}} \frac{1}{1 + \frac{A_{10}}{n_c k_{10}}}$$

$$= \frac{k_{01}}{k_{10}} \frac{1}{1 + \frac{n_{cr}}{n_c}} \quad \text{where } n_{cr} = \frac{A_{10}}{k_{10}}$$

$n_c \gg n_{cr}$

$$\frac{n_1}{n_0} = \frac{k_{01}}{k_{10}} = \text{Boltzman}$$

$n_c \ll n_{cr}$

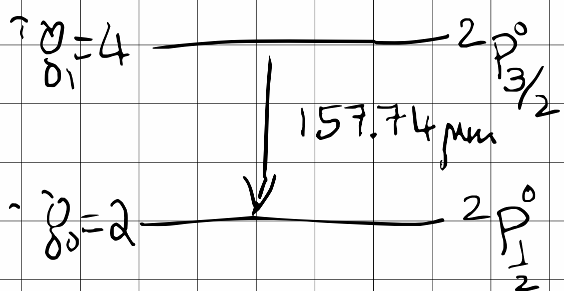
$$\frac{n_1}{n_0} = \frac{k_{01} n_c}{A_{10}} \qquad \boxed{n_1 A_{10} = n_0 k_{01} n_c}$$

In this case every collision leads to a photon being radiated.

In general, can define

$$n_{cr} = \frac{(1+\bar{n}_\gamma) A_{u1}}{k_{u1}}$$

Thus critical density is increased by a background radiation field.



$C^+ \quad 1s^2 2s^2 2p^1$

$A_{10} = 2.4 \times 10^{-6}\ s^{-1}$

$\Omega(^2P^o_{\frac{1}{2}}, {}^2P^o_{3/2}) = 1.6$

$k_{10}(e^-) = 3.4 \times 10^{-8}\ T_4^{-1/2}\ cm^{+3}\ s^{-1}$

$k_{10}(H) = 7.6 \times 10^{-10}\ T_2^{0.1281 + 0.0087 \ln T_2}\ cm^3 s^{-1}$

$n_{crit}(H) \simeq 3.2 \times 10^3\ T_2^{-0.1281}\ cm^{-3}$

$n_{crit}(e^-) \simeq 70\ T_4^{1/2}\ cm^{-3}$

So in CNM, WNM, WIM the densities lie below critical.