

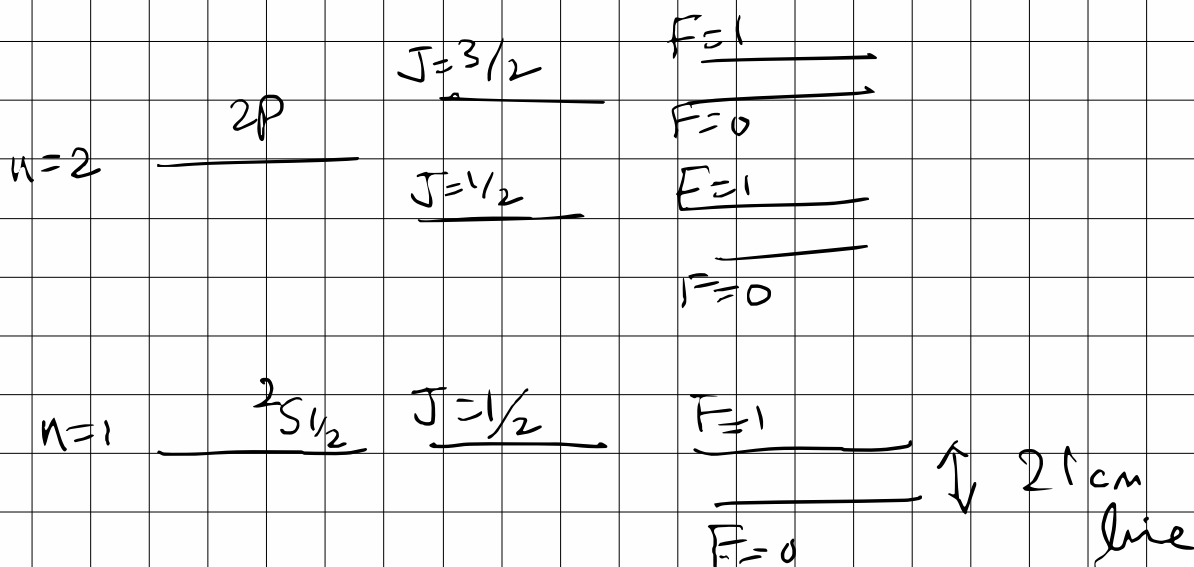
# Hyper-fine Lines

These lines arise from the interaction of the magnetic field of the electron with the magnetic field of the proton.

We introduce a new quantum number

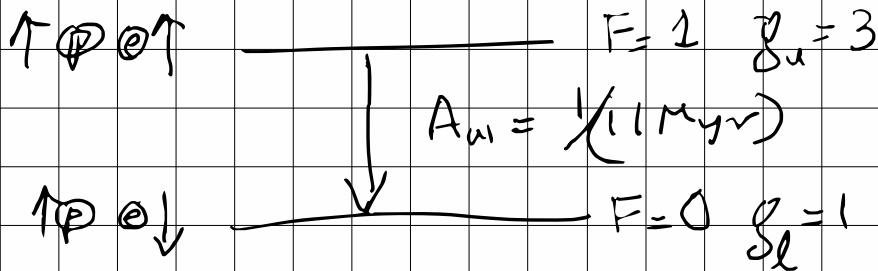
$$\vec{F} = \vec{J} + \vec{I}$$

↑                      ↑  
spint-orbit          nuclear magnetic moment



Recall that the proton and electron have similar angular momenta ( $\hbar$ ) but the proton is  $\frac{m_p}{m_e} = 1800$  times heavier. So the magnetic moment of the proton is much smaller than that of the electron.

For 21-cm line we can use brightness temperature instead of specific intensity



$$E_{ul} = 5.87 \text{ meV}$$

↑ milli

$$\frac{E_{ul}}{k_B} = 68 \text{ mK}$$

↑ milli

We will assume that the 21-cm line is in thermal equilibrium (kinetic temperature)

$$\frac{n_u/g_u}{n_l/g_l} = \exp\left(-\frac{h\nu_{ul}}{kT}\right)$$

$$= 3 \exp\left(-\frac{6.8 \times 10^{-3}}{T}\right) \approx 3$$

$$n_u = \frac{3}{4} n_{\text{HI}} \quad n_l = \frac{1}{4} n_{\text{HI}}$$

Thus, the emissivity is independent of spin temperature

$$j_\nu = n_u \frac{A_{ul}}{4\pi} h\nu_{ul} \phi(\nu)$$

$$= \frac{3}{4} n_{\text{HI}} A_{ul} h\nu_{ul} \phi(\nu)$$

The absorption coefficient is

$$K_{\nu} = n_e \sigma_{lu} - n_u \sigma_{ul}$$
$$= n_e \frac{g_u}{g_l} \frac{A_{ul}}{8\pi} \lambda_{ul}^2 \Phi_{\nu} \left[ 1 - \frac{n_u/g_u}{n_l/g_l} \right]$$

$$= n_e \frac{g_u}{g_l} \frac{A_{ul}}{8\pi} \lambda_{ul}^2 \Phi_{\nu} \left[ 1 - \exp\left(-\frac{h\nu_{ul}}{kT}\right) \right]$$

$$K_{\nu} \approx \frac{3}{32\pi} A_{ul} \frac{hc \lambda_{ul}}{kT_{spin}} n(\text{HI}) \Phi_{\nu}$$

Thus  $K_{\nu} \propto \frac{1}{T_{spin}}$

$$\tau_{\nu} = 2.19 \left( \frac{N_{\text{HI}}}{10^{21} \text{ cm}^{-2}} \right) \left( \frac{100 \text{ K}}{T_{spin}} \right) \left( \frac{\text{km s}^{-1}}{\sigma_{\nu}} \right) e^{-\frac{u^2}{2\sigma_{\nu}^2}}$$

$$N_{\text{HI}} = \int N_{\text{HI}}(\nu) d\nu$$

Can be strong absorption ("self absorption")