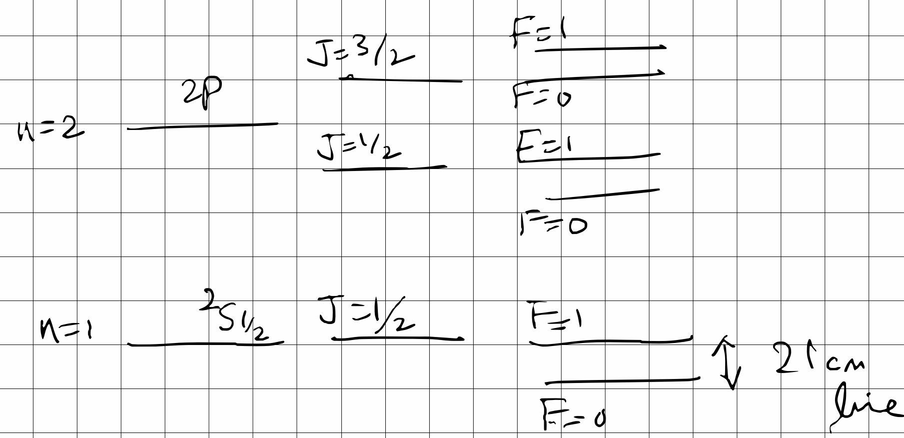
Hyper-fine Lines

These lines arise from the interaction of the magnetic field of the electron with the magnetic field of the proton.

We introduce a new quantum number

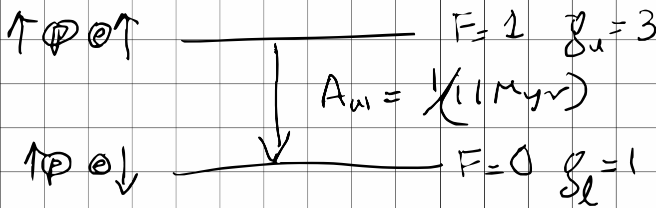
$$\vec{F} = \vec{J} + \vec{I}$$

spin-orbit — nuclear magnetic moment



Recall that the proton and electron have similar angular momenta ($\hbar$) but the proton ~~&~~ is $\frac{m_p}{m_e} \simeq$ 1800 times heavier. So the magnetic moment of the proton is much smaller than that of the electron

For 21-cm line we can use brightness temperature instead of specific intensity



$E_{ul} = 5.87$ meV
← milli

$\frac{E_{ul}}{k_B} = 68$ mK
← milli

We will assume that the 21 cm line is in thermal equilibrium (kinetic temperature)

$$\frac{n_u/g_u}{n_l/g_l} = \exp\left(-\frac{h\nu_{ul}}{kT}\right)$$

$$= 3\exp\left(-\frac{6.8\times10^{-3}}{T}\right) \simeq 3$$

$$n_u = \frac{3}{4} n_{HI} \qquad n_l = \frac{1}{4} n_H$$

Thus, the emissivity is independant of spin temperature

$$j_\nu = n_u \frac{A_{ul}}{4\pi} h\nu_{ul}\,\phi(\nu)$$

$$= \frac{3}{4} n_{HI}\, A_{ul}\, h\nu_{ul}\,\phi(\nu)$$

The absorption coefficient is

$$K_\nu = n_l \sigma_{lu} - n_u \sigma_{ul}$$

$$= n_l \frac{g_u}{g_l} \frac{A_{ul}}{8\pi} \lambda_{ul}^2 \phi_\nu \left[1 - \frac{n_u/g_u}{n_l/g_l}\right]$$

$$= n_l \frac{g_u}{g_l} \frac{A_{ul}}{8\pi} \lambda_{ul}^2 \phi_\nu \left[1 - \exp - \frac{h\nu_{ul}}{kT}\right]$$

$$K_\nu \simeq \frac{3}{32\pi} A_{ul} \frac{hc\lambda_{ul}}{kT_{spin}} n(HI)\, \phi_\nu$$

Thus $K_\nu \propto \frac{1}{T_{spin}}$

$$\tau_\nu = 2.19 \left(\frac{N_{HI}}{10^{21}\,cm^{-2}}\right)\left(\frac{100K}{T_{spin}}\right)\left(\frac{km\,s^{-1}}{\sigma_v}\right) e^{-\frac{u^2}{2\sigma_v^2}}$$

$$N_{HI} = \int N_{HI}(v)\,dv$$

Can be strong absorption ("self absorption")