Ay 215: Homework 1

S. R. Kulkarni

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1 End state of CVs (Pedagogoical exercise)

It is now clear that CVs which are not polars are absent in the orbital period range of 147–191 minutes (Scrhreiber, Belloni & Schwope (2024). Read Paczynski (1981) for a physical understanding of the orbital evolution of CVs. Peruse Knigge, Baraffe & Patterson (2011) for an in-depth version of the same.

We start with an unmagnetized white dwarf of mass, $0.6\,M_\odot$ orbiting a secondary with mass M_2 (circular orbit). Owing to the loss of angular momentum via magnetized winds the secondary is transferring matter to the primary. Our exercise starts when the orbital period reaches 191 minutes. Angular momentum loss via stellar wind ceases. The stellar surface starts to recede from its Roche lobe. The system now enters a phase of CV hibernation. Angular momentum continues to be loss, albeit at a lower rate, via gravitational wave radiation. The orbital period decreases. The stellar surface reaches the Roche lobe surface when the orbital period becomes 147 minutes and mass transfer resumes.

The purpose of this *pedagogical exercise* is to understand the physics of orbital evolution. We do so by re-doing Paczynski (1981). Specifically, derive the orbital period (P_b) and mass of secondary (M_2) curve in the two limits: when $\zeta_{\rm donor}$ is 0.83 and $\zeta_{\rm donor} = -1/3$; here $\zeta_{\rm donor} = d{\rm ln}R_2/d{\rm ln}M_2$ and $\zeta_L = d{\rm ln}R_L/d{\rm ln}M_2$. Next,

1. Compute the mass of the secondary at the start and end of hibernation.
2. Compute ζ_L when the primary angular momentum loss is via GW radiation.

2 Nearest Object

It is quite common in astronomy to deduce considerably from the nearest member. This has proven to be more robust than deducing from the first discovered object. In fact, the first discovered object almost has turned out to be a poor prototype!

Let n_* be the number density of a certain class of stars. Our goal is to compute the probability distribution of the distance to the nearest member of this group, $p_1(r)$. Show

that the cumulative probability of finding the nearest star at distance r is

$$P_1(r) = \int_0^r p_1(r) = 1 - \exp(-\lambda) \quad (1)$$

where $\lambda = (4\pi/3)n_* r^3$. Derive the and plot the cumulative probability function for the 2nd and 3rd nearest stars.

Use: I undertook this exercise when writing a rebuttal paper (Kulkarni & van Kerkwijk) to an alarming paper by Thompson et al. (2009). It is educational and instructive for a student to read these two papers and understand the background.

3 Nearest CVs

List and write down a paragraph on the nearest members of various CV sub-classes (H-rich CV, dwarf novae, novae, recurrent novae, H-poor CV, polar, intermediate polar, super-soft and symbiotic star).

References

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Schreiber, M. R., Belloni, D., Schwope, A. D., The cataclysmic variable orbital period gap: More evident than ever, Astronomy and Astrophysics, 682, L7 (2024)

Thompson, T. A., Kistler, M. D., Stanek, K. Z., A High Rate of White Dwarf-Neutron Star Mergers & Their Transients, arXiv e-prints, arXiv:0912.0009 (2009)