Mass Transfer

S. R. Kulkarni

April 28, 2026

I will adopt the following terminology: the donor star is star 2 with mass M_2. The mass ratio is $q \equiv M_2/M_1$. Let $\zeta_{\dot{M}_2} = d\ln R_*/d\ln M_2$ be the exponent for a secondary star undergoing mass loss where R_* is the radius of the star.

As explained in the class, for stable mass transfer, $\zeta_L \leq \zeta_{\dot{M}_2}$ where $\zeta_L = d\ln R_L/d\ln M_2$ is the exponent for the Roche lobe radius. This quantity is displayed in Figure 1.

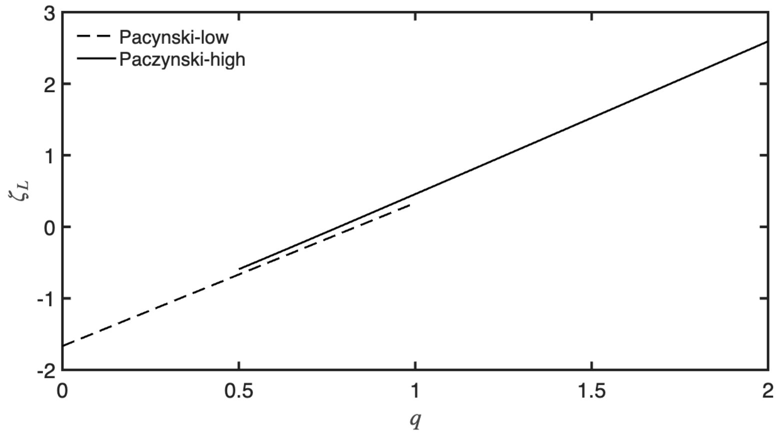

Figure 1: The best linear fit to Paczyśnski-high is $\zeta_L = 2.136q - 1.677$. The fit to Paczński-low is $\zeta_L = 2q - 5/3$. See §A for details.

The value of $\zeta_{\dot{M}_2}$ depends on (1) whether the envelope of the donor star is convective or radiative and the (2) the ordering of mass transfer timescale, $\tau_{\dot{\mathrm{M}}_2} = |\dot{M}_2|/M_2$, relative to two timescales: τ_{dyn}, the dynamical timescale and τ_{KH}, the thermal timescale. The dynamical timescale, computed using dimensional analysis, is $\tau_{\mathrm{dyn}} = 50(\rho/\rho_\odot)^{-1/2}$ minutes where ρ is the mean density of the star. This thermal timescale is the Kelvin-Helmholtz timescale, $\tau_{\mathrm{KH}} = GM^2/(RL) = 31 \times 10^6 (M/M_\odot)^2 (R/R_\odot)^{-1} (L/L_\odot)^{-1}$ yr. Notice the large difference between τ_{dyn} and τ_{KH}. If $\tau_{\dot{M}_2} \lesssim \tau_{\mathrm{dyn}}$ then there is not enough time for heat to flow and so the process can be regarded as adiabatic and the corresponding exponent is $\zeta_{\rm ad}$. If $\tau_{\dot{M}_2} \gtrsim \tau_{\rm KH}$ then star has enough time to respond to mass loss whilst keeping thermal balance. The appropriate exponent is $\zeta_{\rm eq}$. These exponents are discussed in the next section.

1 Adiabatic and Thermal Equilibrium Exponents

1.1 Adiabatic

$$\zeta_{\rm ad} \;=\; \frac{d{\rm ln}R}{d{\rm ln}M},\; R: \text{ mass transfer whilst maintaining the star in hydrostatic equilibrium}$$

The pressure gradient of a star whose envelope has been abruptly removed will be out of equilibrium with the gradient in gravitational potential. Hydrostatic equilibrium will be restored on dynamical timescale (essential the radius of the star divided by the escape velocity).

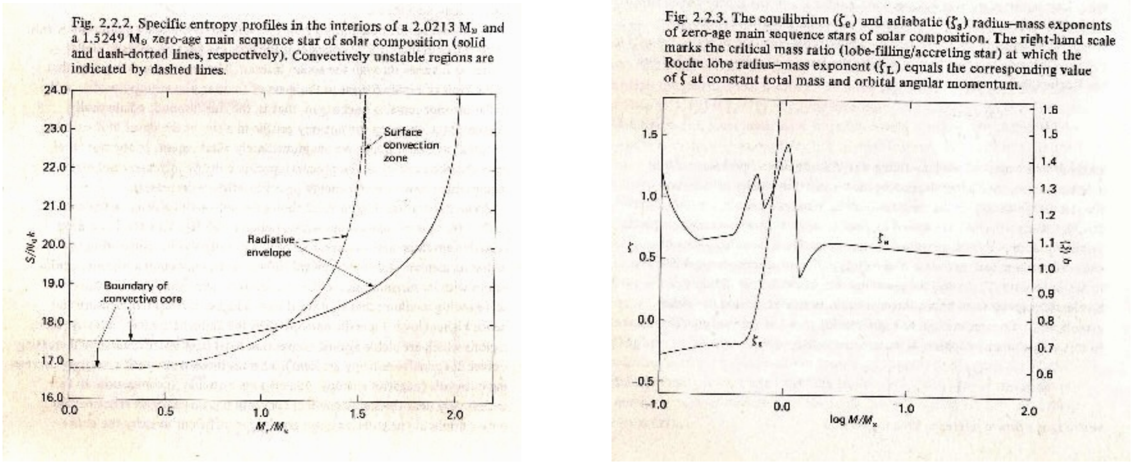
Fig. 2.2.2. Specific entropy profiles in the interiors of a 2.0213 $M_\odot$ and a 1.5249 $M_\odot$ zero-age main sequence star of solar composition (solid and dash-dotted lines, respectively). Convectively unstable regions are indicated by dashed lines.

Fig. 2.2.3. The equilibrium (ζ_e) and adiabatic (ζ_s) radius–mass exponents of zero-age main sequence stars of solar composition. The right-hand scale marks the critical mass ratio (lobe-filling/accreting star) at which the Roche lobe radius–mass exponent (ζ_L) equals the corresponding value of ζ at constant total mass and orbital angular momentum.

Figure 2: From Chapter 3 of Pringle & Wade (1985). ζ_s in this figure is the same as our $\zeta_{\rm ad}$. ζ_e in this figure is the same as our $\zeta_{\rm eq}$.

Convective stars and stars supported by degeneracy pressure are described by the same polytropic index. In both these stars, $P \propto \rho^\gamma$ with $\gamma = 5/3$. The index is usually written as $1 + 1/n$ where n is the "polytropic" index.[1] So, the polytropic index for convective and degenerate stars is $n = 3/2$. The radial temperature of convective stars is very close to the adiabatic gradient. $\zeta_{\rm ad}$ for convective and degenerate stars is $-1/3$.

---

[1] The Lane-Emden equation has analytical solutions for $n = 0, 1, 5$.

Heat transport in radiative stars is resisted by opacity of matter. Opacity depends in a complicated way on density and temperature. For this reason radiative stars do not have a simple radial temperature profile. The adiabatic index, $\zeta_{\rm ad}$ for radiative stars has to be determined numerically.

An extensive discussion of $\zeta_{\rm ad}$ for stars from $0.1\,M_\odot$ to $100\,M_\odot$ can be found in Ge et al. (2015). $\zeta_{\rm ad}$ for several stars on the upper main sequence, computed at various points from ZAMS (top entry) to the base of the red giant phase (bottom entry), are presented in the table below. Notice that for stars in the upper main sequence $\zeta_{\rm ad}$ is large, over most of the life of the stars.

`Table 1: adiabatic exponent for radiative stars (Ge et al. 2015)`

2.5 Msun	5.0 Msun	8.0 Msun
4.126	3.406	2.919
4.498	3.739	3.234
4.879	4.069	3.588
5.227	4.463	3.963
5.659	4.869	4.358
6.097	5.337	4.773
6.594	5.763	5.270
6.817	6.120	5.596
6.880	6.195	5.670
6.993	6.794	6.169
7.823	7.297	6.844
8.189	7.744	7.524
8.486	8.123	8.053
8.582	8.384	8.472
6.838	8.535	8.786
1.515	8.611	9.026
0.722	8.677	9.321
	8.747	9.816
	2.008	10.140
		2.485

1.2 Thermal Equilibrium

$$\zeta_{\rm eq} \;=\; \frac{d{\rm ln}R_{\rm eq}}{d{\rm ln}M},\ R_{\rm eq}\text{: removal of mass on timescales whilst star remains is in thermal equilbrium}$$

Webbink (Chapter 3 of Pringle & Wade 1985) provides an illustrative example to understand why radiative stars shrink rapidly and also the concept of thermal timescale. In

Figure 2 Webbink displays the run of entropy with radius for star of mass $2.02\,M_{\odot}$ and $1.53\,M_{\odot}$. Both have convective cores and radiative envelopes. What happens if we abruptly removed star from the massive star so that its mass is the same as the less massive star? Then in the outer region the truncated star has lower entropy than the less massive star while in the core it is the opposite case. Lower entropy means that the gas is cold and dense with smaller radius and smaller luminosity. The reduced weight on the core turns off nuclear fusion which is only restored after the star has reduced its radius. The star evolves to the entropy configuration of the lower mass star over thermal timescale.

Unperturbed stars on the main sequence are in thermal equilibrium. $\zeta_{\rm eq} \approx 1$ is the mass-radius exponent – well known to neophytes of stellar astronomy courses.

1.3 A global view

Stars on the lower main sequence ($\lesssim 1\,M_{\odot}$) become increasingly convective while those on the upper main sequence become radiative. In the right panel of Figure 2 you can see that $\zeta_{\rm ad}$ is $-1/3$ for low mass stars but rises rapidly as the star develops a radiative envelope. $\zeta_{\rm eq} \approx 1$ over a wide range of masses. However, it has a complex behavior for stars around $1\,M_{\odot}$. This is due to the presence of shallow convective outer layer (cf. the outermost layer of the sun is convective – solar granules!).

2 Various Cases

We are now ready to consider several types of mass transfer. To start with stable mass transfer requires $\zeta_* > \zeta_L$ where ζ_* is the exponent derived for the donor star (which can have a convective envelope or a radiative envelope) and relevant physical process for mass transfer (adaibatic versus thermal equillbrium). So, for instance, $\zeta_L > \zeta_{\rm ad}$, will clearly be unstable.

Case: $\boldsymbol{\zeta_{\rm ad} > \zeta_L}$. *Stable Mass Transfer.*

Here, we restrict the discussion to stars with convective envelopes. We review adiabatic mass transfer from radiative donors later in this section. For convective stars, $\zeta_{\rm ad} = -1/3$. Stable mass transfer requires $\zeta_L < -1/3$. From Figure 1 we see that stable mass transfer requires $q < 2/3$. This case is applicable for CVs which have past the minimum period.

Case: $\boldsymbol{\zeta_{\rm eq} > \zeta_L}$. *Stable mass transfer.*

This case is of interest to radiative donors which are transferring matter on timescales longer than Kelvin-Helmholtz timescale. The run of $\zeta_{\rm eq}$ with stellar mass is given in the right panel of Figure 2. Say, $\zeta_{\rm eq} \approx 1$ in which case (see caption to Figure 1), $q \lesssim 1.25$.

Consider a young CV in which the secondary (usually, a K dwarf) has started mass transferring to the white dwarf. At this point, $\tau_{\dot{M}_2} \gg \tau_{\rm KH}$ and so the donor remains in thermal equilibrium. Stable mass transfer (see caption to Figure 1) requires $q \lesssim 1.25$, readily satisfied by CVs. The Kelvin-Helmholtz timescale scales approximately M^{-n} where $n \approx 2.5$ and so eventually, $\tau_{\dot{M}_2} \lesssim \tau_{\rm KH}$ and the secondary reacts adiabatically to mass loss, $\zeta_* = \zeta_{\rm ad}$. In case of CVs, at this point, the whittled down donor is full convective, $\zeta_{\rm ad} \approx -1/3$ and then the previous case applies.

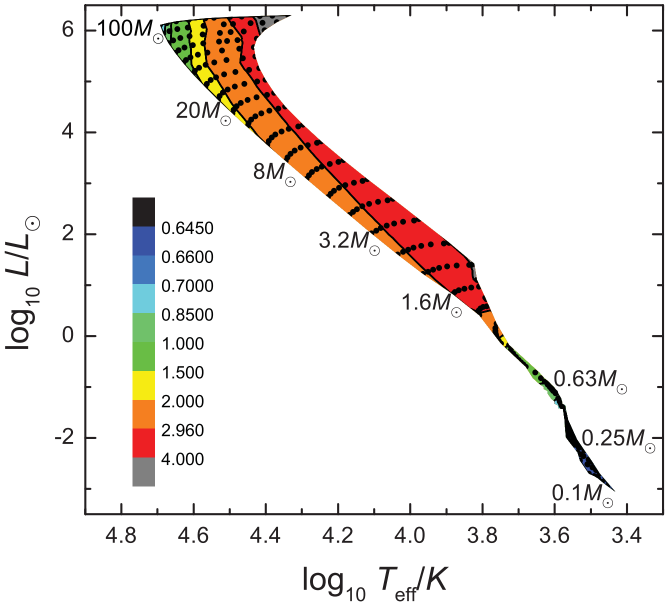

Figure 3: A contour map of the critical mass ratio, $q_{\rm crit}$ (value shown by colored bar), of binary systems with main-sequence donor stars. From Ge et al. (2013).

Case: $\boldsymbol{\zeta_{\rm ad} \geq \zeta_L > \zeta_{\rm eq}}$. *(Stable) Mass transfer at or below thermal timescale.*

This case is of interest to radiative stars losing matter on timescales shorter than the thermal timescale. Consider an extreme example wherein the star loses matter on timescales $\gtrsim \tau_{\rm dyn}$. The large value of $\zeta_{\rm ad}$ means that the star shrinks significantly (on dynamic timescale). As a result, the star loses contact with the Roche surface. Over Kelvin-Helmholtz timescale it expands and at some points starts to transfer matter. If the mass loss rate is very high then the donor will be sent back to the adiabatic phase. The cycle repeats. I suspect that, in steady state, the mass loss rate is determined by the size of the nozzle of the Roche lobe. This transfer rate will be less than the thermal mass loss rate,

$M_2/\tau_{\rm KH}$.

By comparing ζ_L Figure 1 with the table we find that stable mass transfer is possible for values as large as $q_{\rm crit} = 3$ (see also Figure 3 for a full exploration of $q_{\rm crit}$).

Case: $(\zeta_{\rm ad}, \zeta_{\rm eq}) > \zeta_L$. *Stable mass transfer on timescale determined by loss of angular momentum or nuclear evolution.*

In this case, mass transfer is not changing the radius of the star in any significant way (relative to the Roche radius). Mass transfer will only take place if the star grows in size (nuclear evolutionary timescale) or if the orbit is shrunk (due to angular momentum loss).

The master equation now is given by Equation 1. For conservative mass transfer, this equation becomes

$$\begin{aligned}\frac{\dot a}{a} &= 2\frac{\dot J_b}{J_b} - 2\frac{\dot M_2}{M_2}\Big(1-q)\Big) \\ &= 2\Big[(1-q)\tau_{\dot M_2}^{-1} - \tau_{\rm AML}^{-1}\Big]\end{aligned}$$

where $\tau_{\rm AML}^{-1} = -d\ln(J_b)/dt$ and $\tau_{\dot M_2}^{-1} = -d\ln M_2/dt$.

Note that $\dot J_b \le 0$. So, $\tau_{\rm AML}^{-1} \ge 0$.

For $q > 1$ the orbit will always shrink.

For $q < 1$, the orbit will expand if the second term on the RHS does not overwhelm the first term. In this case, for mass transfer to take place the secondary has to increase in size. This is indeed the case with long period LMXBs, the progenitors to long orbital period millisecond pulsars.

So far we have blithely assumed that the donor can arbitrarily lose any amount of matter. The rate at which matter can be transferred is determined by the "nozzle" at L1 and the degree to which the star is overfilling its Roche lobe. However, that is a separate topic and will be discussed in the class today.

3 An example

See Figure 4.

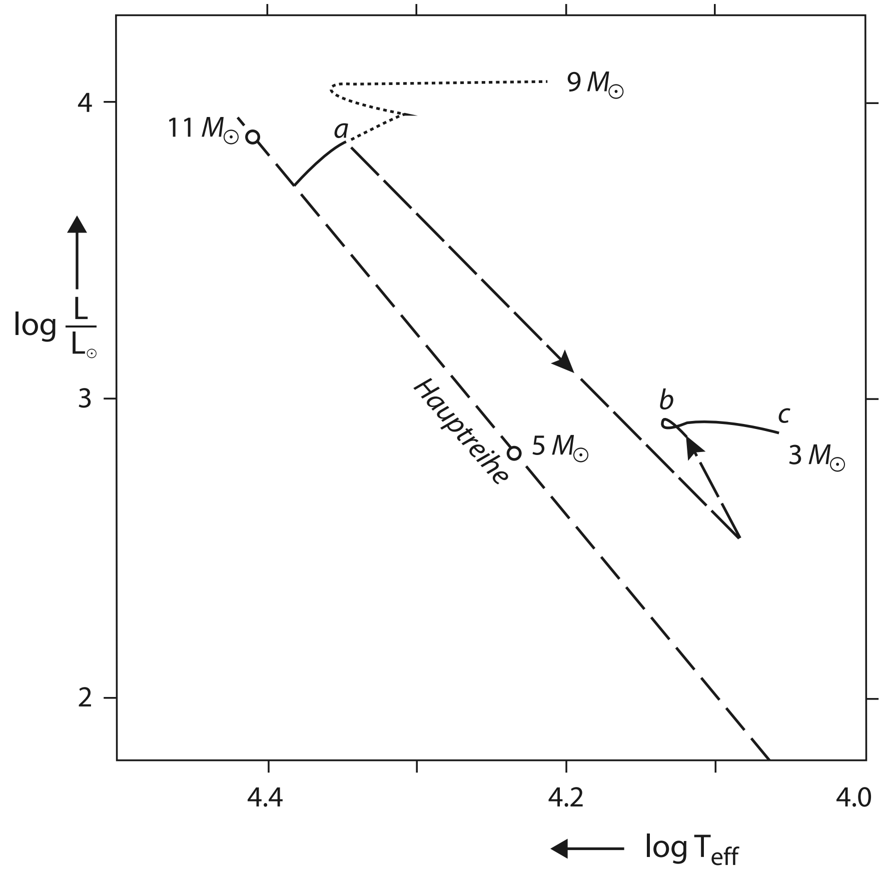

Figure 4: The evolution of a $9\,M_\odot$ & $5\,M_\odot$ star with initial period of 1.2 day (Figure 9.8 of T. Tauri & vdH) . Mass transfer from the primary to the secondary begins at "a". Over thermal timescale it transfers $5.27\,M_\odot$ to the secondary. At point "b" mass transfer stops and the $3.73\,M_\odot$ now evolves as a sub-giant. It transfers matter to the secondary on nuclear timescale. The mass is reduced to $3.02\,M_\odot$ by the time the calculations were terminated. Notice that the secondary, at the end of this mass transfer phase, appears as an ordinary main sequence star of about $11\,M_\odot$ (and noted in the figure). ["Hauptreihe" is main sequence in German].

References

Ge, H., Webbink, R. F., et al., The criteria for dynamical mass transfer of the main-sequence donor stars, Feeding Compact Objects: Accretion on All Scales, 290, 213 (2013)

Ge, H., Webbink, R. F., et al., Adiabatic Mass Loss in Binary Stars. II. From Zero-age Main Sequence to the Base of the Giant Branch, The Astrophysical Journal, 812, 40 (2015)

Pringle, J. E., Wade, R. A., Interacting Binary Stars, Cambridge University Press (1985)

Tout, C. A., Aarseth, S. J., et al., Rapid binary star evolution for N-body simulations and population synthesis, Monthly Notices of the Royal Astronomical Society, 291, 732 (1997)

A Roche lobe exponent

Consider a binary system with stars, masses M_1 and M_2. The mass ratio is $q \equiv M_2/M_1$. The orbital angular momentum of star i is $J_i = M_j a_j v_j$ for $i = 1, 2$. Since $M_1 v_1 = M_2 v_2$ we have $J_1 M_1 = J_2 M_2 = \mu J_b$ where

$$J_b = \sqrt{\frac{G M_1^2 M_2^2}{M} a}\ .$$

The log derivative of the above Equation yields

$$2\frac{\dot{J}_b}{J_b} = 2\frac{\dot{M}_1}{M_1} + 2\frac{\dot{M}_2}{M_2} - \frac{\dot{M}}{M} + \frac{\dot{a}}{a} \tag{1}$$

$\dot{J}_b$ is the sum of loss of angular momenta via all channels (mass transfer, radiation of gravitational waves, stellar winds).

Let star 2 be the donor star and R_L be its Roche lobe. The Roche lobe is proportional to a and so we have $f(q) = R_L/a$. Paczyński (1971), in his Annual Reviews article, gives the following these two fitting for $f(q)$,

$$\begin{aligned} f(q) &= 0.38 + 0.20\log(q) \qquad \text{for } 0.523 \le q < 20 && (2)\\ &= 0.462\Big(\frac{q}{1+q}\Big)^{1/3} \qquad \text{for } 0 < q < 0.523 && (3)\end{aligned}$$

(see also T. Tauri & vdH, §4.1). Matlab and Python users: beware. Note that, in the above Equation, as is common in astronomy, log is logarithm to base 10 (and "ln" is logarithm to base e). When differentiating you need ln. Replace log(x) → $\log_{10}$(e)ln(x) = 0.4343ln(x).

We calculate the Roche lobe exponent, $\zeta_L = (\dot{R}_L/R_L)/(\dot{M}_2/M_2)$. We have

$$\frac{\dot{R}_L}{R_L} = \frac{\dot{f}(q)}{f(q)} + \frac{\dot{a}}{a} \ . \tag{4}$$

Eliminating $\dot{a}/a$ from Equations 1 and 4 we obtain

$$\frac{\dot{R}_L}{R_L} = \frac{\dot{f}(q)}{f(q)} + 2\frac{\dot{J}_b}{J_b} - 2\frac{\dot{M}_1}{M_1} - 2\frac{\dot{M}_2}{M_2} + \frac{\dot{M}}{M} \tag{5}$$

This is our master equation.

For conservative mass transfer, $\dot{M}_1 = -\dot{M}_2$, $\dot{M} = 0$. If angular momentum is conserved then, $\dot{J}_b$=0. So, Equation 5 simplifies to

$$\frac{\dot{R}_L}{R_L} = \frac{\dot{f}(q)}{f(q)} + 2\frac{\dot{M}_2}{M_2}\Big(q - 1\Big)$$

For the choice of Equation 3 we find $\dot{f}/f = {}^1\!/\!{}_3(\dot{M}_2/M_2)$ which then leads to

$$\zeta_L = 2q - \frac{5}{3} \tag{6}$$

as the best fit formula for low mass stars. Choosing Equation 3 instead leads to

$$\zeta_L \quad = \quad \frac{0.2287(1+q)}{1 + 0.5263\log(q)} + 2(q-1)$$

for high mass stars. The best fit linear model for the above equation between $q = 0.5$ and $q = 10$ was found to be

$$\zeta_L = 2.14q - 1.68 \ . \tag{7}$$

See, also, discussion following Eq. 32 of Tout et al. (1997).