Black holes with low mass companions

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April 22, 2026–April 28, 2026

Consider a stellar binary with the primary being a black hole of mass M_1 and a low mass secondary star, mass $M_2 \ll 1\,M_\odot$. Here, we are interested in the regime where $q \equiv M_2/M_1 \ll 1$.

The size of the accretion disk is set by the tidal forces that act upon the outer region of the disk. A fitting formula for the disk radius is

$$\frac{R_{\rm disk}}{a} = \frac{0.6}{1+q}, \qquad 0.03 < q < 1$$

while the circularization radius is given by

$$\begin{aligned} \frac{R_{\rm circ}}{a} &= (1+q)[0.5 - 0.227 \log q]^4, && q < 1 \\ &\approx 0.0859 q^{-0.426} && 0.05 < q < 1\ . \end{aligned}$$

(see T. Tauri & vdH, §4.6.2, §4.6.3).

For small values of q, we see that the circularization radius increases rapidly while the disk radius saturates. What happens when $R_{\rm circ} > R_{\rm disk}$? This question was raised by and but not answered in Yungelson et al. (2006). The abstract of that paper concludes with this sentence "Most of the semidetached black-hole binaries are expected to have periods shorter than 2 h. Properties of such binaries, still to be observed, $q < 0.02$, are different from those of their longer period cousins." In the main text of that paper (§3.3) we find "Mass transfer in systems with very low mass ratios is uncertain and they have been left out of the discussion until now... This can efficiently prevent mass being transferred onto the compact object (by analogy with the gaps created by planets in proto-planetary discs or the inner edges of circum-binary discs)."

According to BlackCAT[1], the living catalog of low mass binary black hole systems identified via X-ray novae, the lowest orbital period, P_b, of an Low Mass Black Hole (X-ray novae) is 2.4 hours (see Appendix A).

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[1] `https://www.astro.puc.cl/BlackCAT/`

Using the mass-density relation for main sequence stars the orbital period of a Roche filling secondary is given by $P_b = 9(M/M_\odot)^{3/4}\,\mathrm{hr}$. Thus, an orbital period of 2.4 hours corresponds to a secondary mass of $0.17\,M_\odot$. Say, the black hole primary mass is $5\,M_\odot$. Then $q = 0.034$. From the above Equations, I find $R_{\rm disk}/a = 0.58$ and $R_\circ/a = 0.50$. The same system with a $10\,M_\odot$ black hole or a shorter orbital period system, say, $P_b = 1.5\,\mathrm{hr}$ ($M_2 = 0.09\,M_\odot$) can have $q < 0.02$. In both cases, $R_{\rm circ} > R_{\rm d}$.

In light of the above discussion, I wondered if the apparent cutoff of $P_b \approx 2.4\,\mathrm{hr}$ is due to cessation of mass transfer in very low q systems. This report is my foray in understanding the fate of LMBH with $q < 0.02$.

The structure of this report is as follows. In §1 I summarize the framework to compute the orbital evolution of mass transferring binary stars. To fortify my understanding, in §2, I investigate the outcome of conservative mass transfer. Lacking a definitive understanding of the fate of mass lost by M_2 I derive the orbital evolution by using two plausible prescriptions (§3, §4) for the specific angular momentum of matter lost by M_2. I conclude in §5.

1 Framework

Our interest is on binaries with $q = M_2/M_1 \ll 1$. The orbital angular momentum of star i is $J_i = M_j a_j v_j$ for $i = 1, 2$. Since $M_1 v_1 = M_2 v_2$ we have $J_1 M_1 = J_2 M_2 = \mu J_b$ where

$$J_b = \sqrt{\frac{GM_1^2 M_2^2}{M} a}\ .$$

The log derivative of the above Equation yields

$$2\frac{\dot{J}_b}{J_b} = 2\frac{\dot{M}_1}{M_1} + 2\frac{\dot{M}_2}{M_2} - \frac{\dot{M}}{M} + \frac{\dot{a}}{a} \tag{1}$$

$\dot{J}_b$ is the sum of loss of angular momenta via all channels (mass transfer, radiation of gravitational waves, stellar winds).

For the radius of the Roche lobe of the secondary star we will use Paczyński's fitting formula,

$$\frac{R_L}{a} = \frac{2}{3^{4/3}}\Big(\frac{M_2}{M_1 + M_2}\Big)^{1/3}\ .$$

The log derivative is

$$\frac{\dot{R}_L}{R_L} = \frac{\dot{a}}{a} + \frac{1}{3}\frac{\dot{M}_2}{M_2} - \frac{1}{3}\frac{\dot{M}}{M}\ . \tag{2}$$

We obtain our "master" equation by eliminating $\dot{a}/a$ between Equations 1 and 2:

$$\boxed{\frac{\dot{R}_L}{R_L} = 2\frac{\dot{J}_b}{J_b} - 2\frac{\dot{M}_1}{M_1} - \frac{5}{3}\frac{\dot{M}_2}{M_2} + \frac{2}{3}\frac{\dot{M}}{M}\ .} \tag{3}$$

Next, define the Roche lobe exponent as, $\zeta_L \equiv \partial\ln R_L/\partial\ln M_2$. Note that $\partial\ln R_L = (\partial\ln R_L/\partial t)dt = (\dot{R}_L/R_L)dt$ and likewise $\partial\ln M_2 = (\partial\ln M_2/\partial t)dt = (\dot{M}_2/M_2)dt$. Thus,

$$\zeta_L = \frac{\dot{R}_L/R_L}{\dot{M}_2/M_2}\ . \tag{4}$$

In a similar fashion, we can define the exponent for the radius of the donor star, $\zeta_{\rm donor} \equiv \partial\ln R_2/\partial\ln M_2$.

As discussed in the class (see T. Tauri & vdH, §4.5.2) stable mass transfer requires that $\zeta_{\rm donor} \geq \zeta_L$. In steady state, the star remains in contact with the surface of the Roche lobe, $R_2 = R_L$. Thus, in steady state, $\zeta_{\rm donor} = \zeta_L$.

2 Conservative Mass transfer

If all of the mass lost by star 2 is accreted by star 1 then $\dot{M}_1 = -\dot{M}_2$. Assume that angular momentum is also conserved, $\dot{J}_b/J_b = 0$. Then application of Equation 1 yields

$$\frac{\dot{a}}{a} = 2\frac{\dot{M}_1}{M_1} + 2\frac{\dot{M}_2}{M_2} = -2\frac{\dot{M}_2}{M_2}\Big(1-q\Big)\ .$$

The above equation informs us that the orbit will expand if $q < 1$, a well known result to tyros of binary stars. The above equation can be integrated to show that $a(t)M_1(t)^2M_2(t)^2$, in this framework, remains constant.

Equation 3 yields

$$\frac{\dot{R}_L}{R_L} = -2\frac{\dot{M}_2}{M_2}\Big(\frac{5}{6}-q\Big)\ .$$

Next, we allow for loss of angular momentum, say through the radiation of gravitational waves,

$$\frac{\dot{J}_{\rm GW}}{J_b} = -\frac{32G^3}{5c^5}\frac{M_1M_2M}{a^4} = -\frac{32G^3}{5c^5}M_2^3\Big(\frac{q+1}{q^2}\Big)\frac{1}{a^4} \tag{5}$$

(T. Tauri & vdH, p. 454). For stable mass transfer, we require $R_L = R_2$. With this assumption, the master equation (Equation 3) for this mode when combined with Equation 4 yields

$$2\frac{\dot{J}_{\rm GW}}{J_b} = \left[\zeta_{\rm donor} + \frac{5}{3} - 2q\right]\frac{\dot{M}_2}{M_2}\ . \tag{6}$$

The LHS is negative. Since $\dot{M}_2 < 0$, stable master requires that $\zeta_{\rm donor} + 5/3 - 2q \geq 0$ or $q \leq \zeta_{\rm donor}/2 + 5/6$. $\zeta_{\rm donor}$ is ≈ 1 if the mass transfer proceeds on a timescale longer than the thermal (Kelvin-Helmholtz) timescale. If it is faster then the value of $\zeta_{\rm donor}$ depends on the structure of the star: $\zeta_{\rm donor} \approx -1/3$ for convective stars and $\zeta_{\rm donor}$ is considerably larger than one for radiative stars. We see that for low q systems the inequality does not pose a problem.

Incidentally, it is the loss of angular momentum via gravitational wave radiation that is driving mass transfer. We can define an efficiency factor for this process by the ratio of the two rates:

$$\eta \equiv \frac{(\dot{M}_2/M_2)}{(\dot{J}_b/J_b)} = (\zeta_{\rm donor}/2 + 5/6 - q)^{-1}\ . \tag{7}$$

For convective stars and degenerate stars undergoing mass transfer on timescale shorter than the Kelvin-Helmholtz timescale, $\zeta_{\rm donor} = -1/3$. For low q systems, this efficiency factor is 3/2.

3 Mass lost via wind

In this case, $\dot{M}_1 = 0$. We postulate that the material lost from star 2 carrying the specific angular momentum of star 2 is ejected from the system. In other words, $dJ_w = (J_2/M_2)dM_2$. Since $J_2M_2 = \mu J_b$ we find

$$\frac{\dot{J}_w}{J_b} = \frac{\dot{M}_2}{M_2}\Big(\frac{M_1}{M}\Big) = \frac{\dot{M}_2}{M_2}\Big(\frac{1}{1+q}\Big)\ . \tag{8}$$

Here, "w" stands for wind. It is instructive to review the evolution of the orbital separation in this framework. Substituting Equation 8 into Equation 1 we find

$$\frac{\dot{a}}{a} = -\frac{q}{1+q}\frac{\dot{M}_2}{M_2} = -\frac{\dot{M}_2}{M}\ . \tag{9}$$

Note that $\dot{M}_2 < 0$ and so, *even in the absence of loss due to angular momentum, the orbit expands, regardless of the value of* q. Since $\dot{M}_2 = \dot{M}$, the above equation can be integrated to yield $a(t)M(t)$ is constant. Thus, as M decreases (due to onset of wind), a increases. This result can also be inferred by noting that the decreased mass leads to a decrease in the gravitational potential energy which then leads to a decrease in kinetic energy (virial theorem) which, in turn, is made possible by the expansion of the orbit.

Now let us include loss of angular momentum via radiation of gravitational waves. Substituting $\dot{J}_w$ (Equation 8) into equation 3 yields.

$$\frac{\dot{R}_L}{R_L} = 2\frac{\dot{J}_{\rm GW}}{J_b} + \frac{\dot{M}_2}{M_2}\Big[-\frac{5}{3} + \Big(\frac{2q/3}{1+q}\Big) + \Big(\frac{2}{1+q}\Big)\Big]$$

As before, in steady state we demand $R_2 = R_L$, which then leads to the LHS being replaced by $\zeta_{\rm donor}$. The resulting master equation for this mode is

$$2\frac{\dot{J}_{\rm GW}}{J_b} = \frac{\dot{M}_2}{M_2}\Big[\zeta_{\rm donor} + \frac{5}{3} - \Big(\frac{2q/3+2}{1+q}\Big)\Big] .$$

Following the framework in the pervious section, for stable mass transfer, the square bracket on the RHS of the above equation has to be ≥ 0. positive. Focusing in the limit of small q, we find this leads to $\zeta_{\rm donor} \geq 1/3$. For low mass stars which are fully convective this condition can be satisfied only if the mass transfer timescale is smaller than the Kelvin-Helmholtz timescale. This is not the case. Thus, the net orbital expansion cannot be matched by the expansion of the donor star. The

4 Mass loss from the binary system

Here, we postulate that the mass lost by the donor departs from the binary system carries the specific angular momentum of the binary system, $dJ_s = (J_b/M)dM_2$; here "s" stands for system. Then we have

$$\frac{\dot{J}_s}{J_b} = \frac{\dot{M}_2}{M} = \frac{\dot{M}_2}{M_2}\Big(\frac{q}{1+q}\Big) . \tag{10}$$

Notice, by comparing with the above Equation 10 with Equation 8, this mode is ineffective in removing angular momentum from the binary system. The orbital evolution, obtained by application of Equation 1, is

$$\frac{\dot{a}}{a} = 2\frac{\dot{J}_{\rm GW}}{J_b} + \frac{\dot{M}_2}{M_2}\Big(\frac{q-2}{1+q}\Big) .$$

Thus, in the absence of any additional loss of angular momentum, the orbit will expand as long as $q < 2$ – a condition easily satisfied for the topic under exploration. The evolution of the Roche lobe radius (Equation 2) is given by

$$\frac{\dot{R}_L}{R_L} \quad = \quad \frac{\dot{a}}{a} + \frac{1}{3}\frac{\dot{M}_2}{M_2} - \frac{1}{3}\frac{\dot{M}_2}{M} = \frac{\dot{a}}{a} + \frac{1}{3}\frac{\dot{M}_2}{M_2}\Big(\frac{1}{1+q}\Big) .$$

The master equation (Equation 3) for this mode is

$$2\frac{\dot{J}_{\rm GW}}{J_b} = \left[\zeta_{\rm donor} + \frac{5/3 - q}{1+q}\right]\frac{\dot{M}_2}{M_2} \ . \tag{11}$$

As before, for stable mass transfer, we demand the factor enclosed by the square bracket to be positive. In the limit of small value of q, this demand becomes $\zeta_{\rm donor} + 5/3 > 0$. Low mass stars and brown dwarf companions with $\zeta_{\rm donor} = -1/3$ satisfy this requirement. Compared to the wind mode, in this mode, the loss of angular momentum is not severe. Apparently, the expansion of the orbit can be matched by the expansion of the star. Note the efficiency of angular momentum losses driving mass transfer (cf. see discussion following Equation 7) is about 3/2, about the same as for the conservative mass transfer model.

5 Conclusions

The smallest orbital period of an LMBH is 2.4 hours (see Table A in the Appendix). It could be due to that our sample is not extensive (yet) or it could be that there is some process which prevents systems with orbital periods shorter than this period not to appear in the X-ray sky. It so happens that typical stellar mass black hole masses the the factor $q = M_2/M_1$ for this system is approaching 0.02. For $q < 0.02$ is happens to be the case that the circularization radius is comparable or bigger than the tidal radius. One simple idea is that in such systems the matter lost by the secondary cannot reach the black hole. If so, such low q systems will not enter an X-ray nova phase. I notice that this problem is more acute if the mass of the black hole is high, say $\gtrsim 30\,M_\odot$. In this case, q could approach 0.003. Perhaps, the reasons LMBHs do not have massive black holes is because of their small q!

Given this uncertain situation I have considered three modes of mass loss: (1) conservative mass transfer, (2) the mass lost by the donor is ejected from the binary system carrying the specific angular momentum of the binary system and (3) the matter is ejected from the binary carrying the specific angular momentum of the secondary. In option 1, by construction the mass lost by the donor is accreted by the black hole. The absence of such systems from the X-ray sky must be due to some other reason (e.g., the accretion disk is cold and thus does not suffer from thermal cycles). In option 2, by construction, there is no accretion disk and so the absence of very low q systems from the X-ray sky is easily explained. In both option 1 and 2, in due course, the donor loses matter and the eventual fate is a brown dwarf.

Option 3: I am confused about the outcome.

In order to make further progress we need to understand the accretion physics of such low q systems. The relevant historical papers are Paczyński (1977) and Paploizou & Pringle

(1977). Perhaps this is an excellent problem for earnest students of Ay215.

Separately, I am struck by the fact that only 33 of the 73 currently known LMBHs have orbital periods measured and even some of them are poorly determined. Since the donors tend to be red dwarfs and M dwarfs the study of quiescent LMBHs is a good project for NIR imaging photometers (e.g., Cryoscope).

6 Postscript

I circulated this write up to stellar a few people at Caltech. Kareem El-Badry pointed out that there are accreting white dwarf binaries (AM CVns) with low q: specifically, V396 Hya ($q = 0.010 - 0.018$; Kupfer et al. 2016) and SDSS J1505+0659 ($q = 0.011^{+0.003}_{-0.011}$; Green et al. 2020).

References

Green, M. J., Marsh, T. R., et al., Spectroscopic and photometric periods of six ultracompact accreting binaries, Monthly Notices of the Royal Astronomical Society, 496, 1243 (2020)

Kupfer, T., Steeghs, D., et al., UVES and X-Shooter spectroscopy of the emission line AM CVn systems GP Com and V396 Hya, Monthly Notices of the Royal Astronomical Society, 457, 1828 (2016)

Paczyński, B., A model of accretion disks in close binaries., The Astrophysical Journal, 216, 822 (1977)

Papaloizou, J., Pringle, J. E., Tidal torques on accretion discs in close binary systems., Monthly Notices of the Royal Astronomical Society, 181, 441 (1977)

Yungelson, L. R., Lasota, J.-P., et al., The origin and fate of short-period low-mass black-hole binaries, Astronomy and Astrophysics, 454, 559 (2006)

A Orbital Periods of LMBH from BlackCAT

Table 1. Orbital periods of LMBH

index	$l(^\circ)$	$b(^\circ)$	P_b (hr)
71	8.6	10.3	10.804 ± 0.001
70	1.1	-3.7	7.0 ± 0.2
69	151.2	5.3	< 8.2
63	35.9	10.2	16.45180.0002
58	358.7	-0.7	$\sim 6.284 \pm 0.001$
55	2.1	1.4	≤ 21
54	29.9	-6.8	> 6.2
53	304.2	-7.6	9.5 ± 0.1
52	13.9	-5.4	< 4.9
50	328.7	50.0	2.8 ± 0.3
49	**5.5**	**16.5**	$\mathbf{2.414 \pm 0.005}$
48	6.4	2.1	< 7
39	24.9	12.2	3.26 ± 0.02
33	336.7	-3.4	7.69 ± 0.02
32	157.7	62.3	$4.0784088 \pm 5 \times 10^{-7}$
31	54.0	8.6	6.58 ± 0.05
30	6.8	-4.8	67.6152 ± 0.0002
27	325.9	-1.8	37.0088 ± 0.0001
21	345.0	2.5	62.926272 ± 0.000096
20	0.1	7.0	6.7 ± 0.2
19	275.9	9.3	6.8449 ± 0.0003
18	45.4	-0.2	812 ± 4
17	165.9	-11.9	5.091850 ± 0.000005
16	295.3	$-0.7.1$	$10.38252 \pm 2 \times 10^{-5}$
15	73.1	-2.1	$155.30803 \pm 5 \times 10^{-5}$
13	63.4	-3.0	8.258095 ± 0.000005
12	310.0	-2.8	61.068 ± 0.002
9	358.6	9.1	12.51 ± 0.03
7	210.0	-6.5	$7.752340 \pm 2 \times 10^{-6}$
6	320.3	-4.4	6.2 ± 0.2
5	338.9	-4.3	42.21 ± 0.01
4	357.2	-4.9	~ 4.4
3	330.9	5.4	26.79377 ± 0.00007

Note. — Column 1: BlackCAT index. Column 2: l. Column 3: b. Column 4: Orbital period. The shortest orbital period is marked by bold face.