Stellar black holes in globular clusters

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FOLLOWING the discovery of X-ray sources in globular clusters, the accretion of matter onto a central massive black hole was suggested[1–3] as a possible explanation. Subsequently, it was found[4,5] that these sources could be readily explained by thermonuclear instabilities on neutron-star surfaces and the black-hole models were abandoned. We show here, however, that the recent discovery[6] of large populations of millisecond pulsars—and hence neutron stars—in globular clusters implies that several hundred stellar black holes (of about ten solar masses) should form within a typical cluster. In clusters of high central density, we find that the rapid dynamical evolution of the black-hole population will cause ejection of nearly all of the holes on a relatively short timescale. But in systems of intermediate density, some of the surviving holes may capture a normal star to form a low-mass X-ray binary. We suggest that there may be one or more such binaries in the globular clusters surrounding our Galaxy. These systems will be quiescent most of the time—with only occasional X-ray outbursts—but future observations of the hard X-ray spectrum may indirectly establish their existence.

Detailed dynamical studies[7] of the rich cluster M15, perhaps the best observed globular cluster, find evidence for $4\times10^4\ M_\odot$ (where $M_\odot$ is the mass of the Sun) of dark matter. This dark matter is attributed to massive white dwarfs, the stellar remnants of main-sequence stars with mass $M \gtrsim 3\ M_\odot$, and neutron stars, the remnants of stars with mass $M > M_N = 8\ M_\odot$. Phinney[8] finds that the data are well fitted with a Salpeter initial mas function (IMF), $dN/dM \propto M^{-\alpha}$ with $\alpha \sim 2.35$. The predicted number of neutron stars at early times is $N_n^i \sim 10^4$. It is an observed fact, however, that neutron stars tend to have large velocities at formation, so we expect only a fraction, $f_N \sim 0.3$, to be retained by the shallow gravitational potential well of a typical globular cluster[9].

Such a large estimate for the present number of neutron stars in clusters, N_n is consistent with the huge abundance of cluster pulsars[6]. Moreover, the radio pulsar observations also show that N_n depends only on gross structural parameters (total mass, core density). No apparent dependence has been found either on the metallicity of the cluster (for example M15, in which at least eight pulsars[10] have been found, is metal poor, whereas 47 Tuc, with at least ten known pulsars[11], is metal-rich), or on the value of α for the currently visible stars with $M \lesssim 0.8 M_\odot$. Pulsars have also been found in moderate- and low-density clusters such as M13 and M53.

We assume that the cluster pulsars are neutron stars, the stellar remnants of massive stars, spun up by accretion of matter from a companion (that is during the low-mass X-ray binary (LMXB) phase)—the usual model for millisecond pulsars. Others[12] argue that the Salpeter IMF will result in cluster disruption (due to rapid mass loss from stellar evolution), and have speculated that all millisecond pulsars are accretion-induced transmuted white dwarfs born with low velocities. The mass of such neutron stars, however, would be $1.25\ M_\odot$, whereas the best-measured cluster neutron stars have masses similar[13] to 'ordinary' pulsars. In addition, the scale height of the observed millisecond pulsars in the disk of the Galaxy is consistent with large space motions for such objects. We therefore favour the standard model, and now consider the logical extension of this hypothesis—black holes, the stellar remnants of even more massive stars, with $M > M_B$.

The value of M_B, unlike M_N, is not well known. Evolutionary models and statistics of Galactic black-hole systems suggest[14] $M_B \sim 60 \pm 20\ M_\odot$. Models[15] of chemical evolution of galaxies at early times, and hence with metallicities representative of protoglobular-clusters, require stars above $25\ M_\odot$ to collapse to black holes without ejecting processed material. The precise mass of the resulting black holes, m_b, depends upon the mass-loss history. For the Galactic binaries, we find estimates of m_b to be $16\ M_\odot$ (Cyg X-1; ref. 16), $12\ M_\odot$ (GS 2023+338; ref. 17), $6\ M_\odot$ (Nova Muscae; ref. 18), $7\ M_\odot$ (A0620-00; ref. 16) and $7\ M_\odot$ (LMC X-3; ref. 16). We adopt $m_b = 10\ M_\odot$. This assumption contrasts with an earlier study[19] which considered the structural influence of a substantial hypothetical population of low-mass ($\lesssim 2.5\ M_\odot$) black holes.

In the absence of any compelling evidence to the contrary, we make the reasonable assumption that the IMF for stars with $M \gtrsim M_N$ follows a Salpeter distribution, as appears to be the case for M15. With this assumption, and starting with a conservative estimate $N_n^i \sim 3\times10^3$, we find $N_b \sim 150$–400, for the above range in M_B. Unlike the neutron stars, the black-hole X-ray binaries in the Galactic disk do not show peculiar radial motion as large as LMXBs (S.R.K., manuscript in preparation). Consequently, we assume that the retention factor $f_B \lesssim 1$. We will start with the conservative assumption that rich clusters began with $N_b \sim 10^2$ black holes of mass $m_b = 10\ M_\odot$, and consider their fate as the cluster moves inexorably toward core collapse[20,21]. Later we will indicate how the evolution of the black-hole subsystem changes as N_b increases.

Once formed, the black holes soon come into energy equipartion with the typical star (of mass $m \sim 1\ M_\odot$), slow down, and settle to the centre of the cluster. This process of mass segregation takes place on the dynamical-friction time scale, $\sim t_{Rh}/\mu \sim 0.1 t_{Rh}$, where $\mu = m_b/m$, and t_{Rh} ($\sim 10^9$ yr typically) is the cluster half-mass relaxation (thermalization) time[20]. The process is complete by the time the cluster is $\sim 10^9$ yr old, and most of the holes are in equipartition in the core. Because the holes constitute only a small fraction of the total mass of the cluster, they do not greatly affect its overall dynamical evolution. Equipartition in the harmonic potential determined by the core stars gives $r_b^2/r_c^2 \sim v_b^2/v_c^2$, where r_b and v_b are the half-mass radius (that is, the radius enclosing the inner half of the cluster mass) and velocity dispersion, respectively, of the black holes, and r_c and v_c are core radius and velocity dispersion of the stars. Thus the holes become concentrated in the core, with $r_b/r_c \sim \mu^{-1/2} \sim 1/3$.

The long-term evolution of the system is driven by two-body stellar encounters, which allow individual stars to exchange energy (on a relaxation time scale), and provide a mechanism for heat conduction from the core of the cluster to its periphery. As a result, the core collapses—both r_c and N_c, the number of stars in the core, decrease (formally) to zero in $\sim 10 t_{Rh}$ (ref. 20). During this process, the central regions can be reasonably approximated as an isothermal sphere, and the central stellar mass density increases roughly as $\rho_c \propto r_c^{-2} \propto N_c^{-2}$. However, the central mass density of the holes increases faster, $\propto r_b^{-3} \propto r_c^{-3}$, so long as the number of holes is conserved. The ratio of the central density of the holes to that of the stars is

$$\rho_b/\rho_c \sim \mu (N_b/N_c)(r_c/r_b)^3 \sim \mu^{5/2}(N_b/N_c)$$

Therefore, if the holes did not interact with one another, their central density would become comparable to that of the stars by the time the core had shrunk to contain $N_c \sim N_b \mu^{5/2} \sim 3\times10^4$ stars. At around this time, the Spitzer mass-segregation instability[20]—the runaway collapse of the black-hole subsystem when it decouples from the stellar core—would be expected to set in.

The holes do interact with each other however, forming hole–hole binaries which subsequently eject single holes from the cluster. The timescale for 'three-body' hole–hole binary-formation (in which a third hole carries away the excess energy required for two others to become bound) is[22]

$$t_{3B} \sim 3\mu^{-25/2} N_b^{-3} N_c^5 \tau_c$$

where $\tau_c = r_c^{3/2}/(GN_c m)^{1/2}$ is the time required for a typical star to traverse the core. We assume that this process starts to become important when $t_{3B} \sim 0.1 t_{cc} \sim 5 t_{Rc}$ (the coefficient depending somewhat on the evolutionary state of the core), with t_{cc} the time left until core collapse (that is, when $r_c = 0$) and t_{Rc} the two-body relaxation time for the core stars:

$$t_{Rc} \sim 0.1 N_c \tau_c$$

In this expression, we have taken the usual 'Coulomb' logarithmic factor[20] to be ~ 10. Equating t_{3B} and $0.1 t_{cc}$, we obtain:

$$N_c \sim 0.6 \mu^{25/8} N_b^{3/4}$$

For our choice of parameters, we again find $N_c \sim 3 \times 10^4$. For a typical globular cluster, this corresponds to $r_c \sim 1$ pc and $t_{Rc} \sim 2 \times 10^8$ yr. Thus hole–hole binaries begin to form several billion years before core collapse, effectively preventing development of the mass-segregation instability.

For larger values of N_b, both the onset of the mass-segregation instability and the formation of the first black-hole binary occur earlier in the cluster evolution. From the above scaling relations, however, we see that the Spitzer instability is (slightly) favoured as N_b increases. Once this instability sets in, the black-hole subsystem decouples from the stellar core and enters a phase of rapid dynamical evolution, undergoing core collapse on a timescale $\sim t_{Rb} \sim \mu^{-5}(N_c/N_b) t_{Rc} \sim \mu^{-5/2} t_{Rc} \ll t_{cc}$. The resulting high spatial density leads to the almost immediate formation of black-hole binaries once runaway mass segregation begins. The production of binaries continues until N_b falls below the threshold for stability. The remaining few holes must wait for the core to shrink to the point where the previous discussion applies. In our specific estimates below, we will use our initial assumption of $N_b \sim 10^2$. Also, we will focus on stellar-mass black holes, that is, we will not discuss the possibility that a post-collapse runaway merging of normal stars will produce a single massive black hole[23].

Once formed, a black-hole binary becomes more tightly bound ('hardens') as a result of superelastic interactions with other holes[24]. Its recoil velocity after each encounter increases, and it is eventually ejected from the cluster. Following Hut *et al.*[25], and taking into account the greater mass of the holes, we estimate the time required for a hole–hole binary to escape from the cluster in this manner to be $\sim 3 \times 10^3 t_{Rb} \sim 10 t_{Rc}$ (a few times 10^8 yr). Based on *N*-body simulations of binaries in small-*N* systems (here, the black-hole subsystem), we expect that at most one or two hole–hole binaries will be present in the cluster at any given time. As each binary ejects ~5–10 single black holes as it hardens, we conclude that the black-hole population will be depleted in less than a billion years, well before core collapse is complete. From the previous discussion, we see that this conclusion is largely independent of the precise initial value of N_b.

We note that superelastic encounters between black-hole binaries and stars may inject a substantial amount of energy into the cluster. The total energy supplied by a single black-hole binary as it hardens from formation at $\sim 1\,kT$ to ejection at $\sim 10^4\,kT$ (where $\frac{3}{2}\,kT$ is the mean stellar kinetic energy) is $\sim 3 \times 10^3\,kT$, or about 5–10% of the core kinetic energy. The bulk of this energy is actually spread over a fairly large fraction of the entire cluster, as reaction products recoil into the halo and the black holes slowly drift back to the centre by dynamical friction. We therefore expect that, while binary heating may significantly slow the overall collapse of the stellar core, it will probably not halt it completely, at least for $N_b \lesssim 10^3$. In any case, these hole–star interactions will have little direct influence on the black-hole subsystem itself, as it evolves primarily by hole–hole encounters.

By the end of the core-collapse phase, it is likely that the last hole–hole binary will still remain in the core, having ejected all other holes. There is a small chance that this final binary will itself have escaped following the interaction that ejected the last single hole. Similarly, there is a small chance that a lone third hole is left on a very wide 'parking' orbit, with its apocenter at several half-mass radii[25]. The ejected black-hole binaries will have orbital separations of ~1 AU. The timescale on which they are expected to coalesce by means of the emission of gravitational radiation is $\sim 10^{14}$ yr, much longer than the age of the Universe. Thus we expect that they will not be significant sources for future gravitational wave interferometers.

We have seen that there is an interval of $\sim 10^9$ yr, probably occurring well before core collapse, when black-hole binaries can form and the hole population is largely depleted. During this period, a few holes may acquire companions through tidal capture, forming close binaries with typical separations of a few (5–10) stellar radii. At early times, the stellar density would not be high enough for this process to be likely; at late times the holes will be ejected from the core (although this will probably leave the hole–star binary intact, as the hole–hole binary will have a size $\sim 10^2$ larger). Although the existence of these binaries is unimportant to the overall dynamics of the cluster, they do provide us with a possible, (if transient), means of observing the black-hole subsystem. The number of black-hole tidal-capture binaries formed in our model cluster is $\sim 0.01 N_b$. Given the measured rate of core collapse[21], we estimate that there are currently $2 \times 0.01 N_b \sim 10$ such binaries in the Galactic globular cluster system.

Mass transfer begins only when the companion evolves onto the subgiant branch. The probability that this will take place for a typical core star during the $\sim 10^9$ yr before the black hole is ejected from the cluster is ~0.1. Therefore, we estimate that the number of active hole-containing low-mass X-ray binaries in the globular cluster system is ~1. Note that our analysis also predicts the presence of several such black-hole binaries in the Galactic bulge and halo, as a result of ejection a few billion years ago.

Like their Galactic disk counterparts (for example A0620-00, ref. 26), the cluster accreting black-hole binaries are expected to be transients. An explanation[27] for this transient behaviour is an accretion disk instability that arises only when $q \equiv M_d/M_a < 0.25$, where M_d and M_a are the masses of the donor and accretor stars. This condition is easily satisfied by the proposed cluster black-hole binaries. The disk black-hole binary transients show hard X-ray tails and it would be desirably to search for (and then monitor) hard X-ray transients with GRANAT, ASCA, the Compton Gamma Ray Observatory and future missions like the X-ray Timing Explorer (especially the All Sky Monitor).

Recently, Brown and Bethe[28] have suggested that main-sequence stars with $18\,M_\odot < M < 25\,M_\odot$ evolve to low-mass ($1.5\,M_\odot$) black holes, and that stars above $25\,M_\odot$ evolve to high-mass ($\gtrsim 10\,M_\odot$) black holes of the sort seen in the Galactic disk. In clusters, such low-mass black holes behave dynamically like neutron stars. Their number is $N_{lmb} \sim N_n f_{lmb}$, where f_{lmb} is the unknown retention factor. Because all cluster LMXBs[29] (save two for which few observations exist) show bursts, we conclude that either $f_{lmb} \ll 1$, or that this mechanism does not operate.

Holes in clusters offer the possibility of a black-hole-millisecond pulsar binary system with large eccentricity and small orbital period, a system ordinarily not possible in the Galactic disk. Such systems can be profitably used for tests of general relativity (akin to studies of binary pulsar systems[30]). Radio pulsar searches that are sensitive to compact binaries, although time consuming, may prove fruitful in the long run. □

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Primordial black holes in globular clusters

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IT HAS recently been recognized[1] that significant numbers of medium-mass black holes (of order 10 solar masses) should form in globular clusters during the early stages of their evolution. Here we explore the dynamical and observational consequences of the presence of such a primordial black-hole population in a globular cluster. The holes initially segregate to the cluster cores, where they form binary and multiple black-hole systems. The subsequent dynamical evolution of the black-hole population ejects most of the holes on a relatively short timescale: a typical cluster will retain between zero and four black holes in its core, and possibly a few black holes in its halo. The presence of binary, triple and quadruple black-hole systems in cluster cores will disrupt main-sequence and giant stellar binaries; this may account for the observed[2] anomalies in the distribution of binaries in globular clusters. Furthermore, tidal interactions between a multiple black-hole system and a red-giant star can remove much of the red giant's stellar envelope, which may explain the puzzling absence[3] of larger red giants in the cores of some very dense clusters.

Theories of the evolution of massive stars suggest that some fraction of the most massive ones will form black holes of $\sim$5–20 $M_\odot$ (ref. 4), where $M_\odot$ is the mass of the Sun. The end product of stellar evolution, whether a neutron star, black hole, or complete disruption of the star, may depend on the rotation of the stellar core and the metallicity of the envelope, hence the fraction of stars that collapse to black holes is not well determined. Models suggest that low metallicity stars may form black holes more readily than stars of solar metallicity[5]. For a range of initial mass functions (IMFs) and stellar masses, estimates suggest that from 10^{-4} to less than 10^{-6} of stars will end as black holes. The same process of stellar evolution should produce black holes in globular clusters[6] (S. Kulkarni, personal communication) with interesting observational and dynamical implications. As globular clusters contain of order 10^6 stars we expect between zero and 100 black holes to form in each cluster. If the IMF is biased towards massive stars at high densities, the fraction of stars leading to black holes may be greater in protocluster cores[1]. Globulars contain a large number of neutron stars[7]. Assuming that most of these neutron stars are 'primordial' and not the product of accretion induced collapse[8]), and as the star formation process is unlikely to 'know' the mass at which evolved stars collapse to black holes (rather than neutron stars) we can have some confidence that the IMF does not steepen sharply at that critical mass, and that black holes are likely to form in protoclusters. Because black holes, unlike neutron stars, are not thought to receive kicks when they are produced[9], and recoil from binary dissociation during black hole formation will be small, primordial black holes ought to be retained in globulars when formed. A dynamically relaxed cluster forms an equilibrium 'core', where the stellar density gradient is small and stars are in kinetic equilibrium, moving with a velocity dispersion $\sim\sigma\times(\bar{m}/m_*)^{1/2}$, where m_* is the mass of a star, moving in a multi-mass field with average stellar mass $\bar{m}$. Typically, $\sigma\sim 5\ \mathrm{km\ s^{-1}}$. Black holes will sink to the core on a dynamical friction timescale, t_{df}

$$t_{\mathrm{df}}\sim 3\times 10^7\left(\frac{\sigma}{10\ \mathrm{km\ s^{-1}}}\right)\left(\frac{r_c}{1\ \mathrm{pc}}\right)\left(\frac{M_{\mathrm{BH}}}{10\ M_\odot}\right)^{-1}\ \mathrm{yr} \qquad (1)$$

where r_c is the cluster core radius and M_{BH} is the mass of the black hole. This process starts after the stellar population has evolved to the point where the main-sequence turnoff is well below 5 $M_\odot$, and all primordial black holes and neutron stars have formed. Typically this will be a few hundred million years after the formation of the cluster.

In the core, the black holes will form binaries. Stars with masses 5–1.4 $M_\odot$ evolve to white dwarfs in a few billion years, when any primordial black holes will be an order of magnitude more massive than the surviving stars. The black holes random-walk (by stellar 'brownian motion') in the core over a radius of order 10^5 AU. Collective effects lead to the black holes becoming bound into black hole–black hole (BHBH) binaries[10]. The binding energy of BHBH binaries exceeds the stellar kinetic energy at semi-major axis ($a\lesssim 10^3\ \mathrm{AU}(M_{\mathrm{BH}}/10\ M_\odot)^2/(\sigma/10\ \mathrm{km\ s^{-1}})^2$) and the black hole binaries are hardened by encounters with stars and other black holes; that is a shrinks and the binary binding energy increases. If the number of black holes in the core is large (>4), the black holes and BHBH binaries interact. A strong BH–BHBH encounter releases about 1/3 of the BHBH binding energy[11]. This energy is released as kinetic energy of the bodies, with each component recoiling at a speed inversely proportional to its mass. A BHBH binary can recoil from the core once the binaries have hardened to semi-major axis of $\sim$30 AU. Black holes and BHBH binaries ejected from the core sink back because of dynamical friction and continue interacting. At $\lesssim$10 AU, BH–BHBH encounters eject the black holes from the cluster. This process occurs on a timescale $\sim 10^8\ \mathrm{yr}/[(a/10^3\ \mathrm{AU})(n_{\mathrm{BH}}/10^4\ \mathrm{pc^{-3}})]$ (refs 11, 12), where n_{BH} is the number density of black holes in the core. (Because dynamical friction will bring the black holes into the inner 0.3 pc in all but the lowest density globulars, $n_{\mathrm{BH}}\gtrsim 10^2$ after the initial relaxation of the black-hole population in a young cluster rich in black holes.) Independent of the initial number we expect most of the black holes to be ejected from the cluster on a timescale short compared to the cluster lifetime. The final black-hole population will depend on the random encounters of the last one or two black-hole binaries and whether these provide sufficient recoil to eject the black holes from the cluster, or whether the black holes end up on wide orbits beyond the cluster half-mass radius but bound to the cluster. We do not expect all clusters to contain black holes at this stage. In order to estimate the fraction of clusters that eject all their black holes, it will be necessary to perform