FOLDING A TIME SERIES

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Consider a period signal, period P, sampled every Δt: $y(k)$ where $k = 0, 1, 2, ..., n-1$. The duration of the input time series is thus $T = n\Delta t$.

The simplest way to find if the time series is periodic is to Fourier transform the data, $Y(f)$.[1] The power spectrum is $P(l) = Y(l)Y^*(l)$. $P(l)$ exhibits a spike at the first harmonic (in music theory this is called as the fundamental), the second harmonic and so on. Let the corresponding Fourier bins be l_1, l_2, l_3 and so on.

As I explained in the class, folding the time series to the correct period will (1) recover the pulse profile and (2) the signal-to-noise-ratio (SNR) of the profile will be much higher than the SNR of each input data point.

In order to understand the concept of "folding" the time series you need to understand the concept of phase. The phase is best visualized as the azimuthal angle of a rotating object. As with angles, you only keep angles less than 360°. Instead of 360° we use phase in which one rotation is one unit. As with all rotations or angles we only keep the fractional part.

The phase corresponding to $y(k)$ is

$$\phi_k = \left[\frac{k\Delta t}{P}\right] \tag{1}$$

where $[x] \equiv x - \mathtt{fix}(x)$ where fix(x) is the integer part of x. Note $0 \leq \phi_k < 1$. We agree to display the folded profile over, say, m, bins. The bins thus range from 0,1,2,3..,$m-1$. Define a pulse profile array, $B(0, 1, 2.., m-1)$. Then the phase bin number corresponding to ϕ_k is $q = \mathtt{floor}(m\phi_k)$. Go to this pulsar profile bin, $B(q)$ and add in the value of $y(k)$. Do the same thing for each successive value of y. The result is the pulse profile.

1. Precision of period

Say you have a time series which is T long. What is the best estimate of precision of the period, P? Let P' be the trial period. Then over time the accumulated phase is $\phi' = T/P'$. Compare this to the phase accumulated by the true period, $\phi = T/P$. The difference is then

$$\begin{aligned} \phi' - \phi &= \frac{T}{P'} - \frac{T}{P} \\ \delta\phi &\approx \frac{T}{P}\frac{\Delta P}{P} \end{aligned} \tag{2}$$

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[1]We will discuss Fourier transforms later in the course.

The profile will not be "washed away" if $\delta\phi$ is well below unity. Thus the sensible precision we can demand to know the period is

$$\delta P \lesssim P \frac{P}{T} \delta\phi \tag{3}$$

where you may wish to set $\delta\phi$ to say 0.1. Note that $P/T \approx 1/n$ where n is the number of cycles over the duration of the signal. Thus $\delta P \approx 0.1(P/n)$.