How to find stellar black holes?
(Notes)

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Preface

This are notes I developed whilst teaching a mini-course on "How to find (stellar) black holes?" at the Tokyo Institute of Technology, Japan during the period December 2018–February 2019. This was a pedagogical course and not an advanced research course. Every week, over a one and half hour session, I reviewed a different technique for finding black holes. Each technique is mapped to a chapter.

The student should be aware that I am not an expert in black holes, stellar or otherwise. In fact, along with my student Helen Johnston, I have written precisely one paper on black holes, almost three decades ago. So, I am as much a student as a young person starting his/her PhD. This explains the rather elementary nature of these *notes.* Hopefully, it will be useful introduction and guide for future students interested in this topic.

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• *Chance favors the prepared mind.*
• *The greatest derangement of the mind is to believe in something because one wishes it to be so – L. Pasteur*

• *We have a habit in writing articles published in scientific journals to make the work as finished as possible, to cover all the tracks, to not worry about the blind alleys or to describe how you had the wrong idea first, and so on. So there isn't any place to publish, in a dignified manner, what you actually did in order to get to do the work. – R. P. Feynman*

Guide to Usage

The only way to learn a subject is to work out problem sets. I have given a range of "homeworks". Here is the translation:

Problem is primarily an exercise in pedagogy.
Exercise is also an exercise in pedagogy but requires investment of time (<1 hour). Will feel good after finishing the exercise (like drinking celery juice after a 6 am run).
Literature Survey as the name suggests is aimed at student becoming familiar with a specific topic by surveying the literature. [Use ADS with appropriate key words to find papers. Usually the latest papers and a list ordered by citations are helpful].
Project. Suitable for an undergraduate thesis project. In some cases the suggested project could be the basis of a Master's thesis project. However, check with a knowledgeable colleague before embarking on suggested projects! *Caveat emptor!*.

Nitty gritty:
1. $\log(x)$ is logarithm to base 10 whereas $\ln(x)$ is logarithm to base e.

Contents

Chapter 1

Black Holes

Two types of black holes have been observed: stellar mass black holes ($5\,M_\odot$ – $-80\,M_\odot$) and massive black holes that are found at the centers of galaxies ($10^6\,M_\odot$ to $10^{10}\,M_\odot$). There are persistent claims of intermediate mass black holes ($\sim 10^3\,M_\odot$) but to my knowledge there is no object proven to be so at a high degree of confidence. The distinct mass ranges of the stellar and nuclear black holes strongly point to differing formation paths. There is sound evidence that the stellar mass black holes are the end points of massive stars. There is growing evidence that the "nuclear" black holes have grown by successive cannibalizing episodes. A major question in astronomy is the mass spectrum of the black holes formed in the very early Universe. It is these black holes that are the seeds for nuclear black holes.

Stellar black holes are interesting for many reasons. At the highest level, formation of a stellar black hole is a fundamental phenomenon in astronomy and physics. Even if General Relativity is taken for granted the formation of a black hole involves exquisite understanding of neutrino physics and nuclear physics, in particular the equation of state of dense cold matter. Next, the timescales associated with black holes is linearly proportional to the mass. Accreting stellar black hole binaries are well suited to master's thesis and PhD thesis timescales. Furthermore, their proximity ensures strong signals and so the objects are well suited for detailed studies.

Interest in stellar black holes has been grown exponentially with the commissioning of gravitational wave interferometers: the Laser Interferometer Gravitational-wave Observatory (LIGO) and Virgo. In fact, the field of gravitational wave (GW) astronomy was inaugurated in September of 2015 with the event GW 150914, which resulted from the coalescence of two stellar black holes. This discovery was surprising in two ways. (1) Astronomers, by and large, had expected that the most common GW events would be coalescence of two neutron stars (CNS) and thus likely the first event would be CNS. (2) The inferred masses of $36^{+5.2}_{-3.8}\,M_\odot$ and $29.1^{+3.7}_{-4.4}\,M_\odot$ was considerably than that expected of stellar black holes (identified, until then, via electromagnetic channels). These large masses has led to an explosion of activity by stellar astronomers to make sizable mass stellar black holes.

The old and simple story was that stars above $8\,M_\odot$ formed neutron stars and more massive stars, say those above $20\,M_\odot$ initial mass, formed black holes. However, some developments (gamma-ray bursts, super luminous supernovae) suggest that there are parameters other than the initial mass of the star which determines the *kismet* of massive stars. Complicating factors include metalicity and rotation.

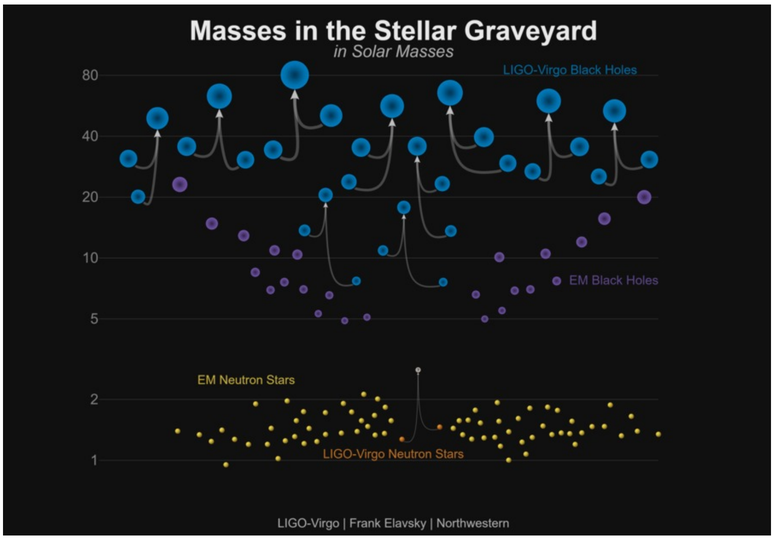

Figure 1.1: Schematic diagram which summarizes our current knowledge of neutron stars and stellar black holes. The mass of the objects can be read off from the vertical axis. The horizontal axis has no special meaning. From LIGO homepage. A pleasant and the same time a productive way to procrastinate is to watch `https://www.ligo.caltech.edu/LA/video/ligo20181203v1` and related videos.

Figure 1.1 is an amazing figure that encapsulates the field of stellar black holes. At the bottom we see that the neutron stars have masses in the range 1–2 $M_\odot$. The binary neutron star coalescence, GW170817, resulted in a black hole of mass $2.82^{+0.47}_{-0.09}\,M_\odot$ (this event is schematically shown at the bottom, about midway). Moving up, the stellar black holes found via electromagnetic methods (and discussed in Figure 1.2) are shown by violet disks. Moving further up, black hole coalescences and detected by LIGO-Virgo are shown in dark blue. The coalescence events are the most violent events in the Universe. For instance, GW150914 radiated about $3\,M_\odot$ of energy with peak luminosity of about $4\times 10^{56}\,\mathrm{erg\,s^{-1}}$!

Clearly, the study of black holes is in a state of ferment and fever. It is thus a timely topic to study! With little doubt GW facilities will drive the field of stellar black holes. LIGO & Virgo will discover large numbers of BH-BH binaries and the analysis of the waveforms outperform all other tests of GR. However, the lack of precise localization of BH mergers means that the GW samples will provide clues via statistical arguments (e.g. the rate of increase with redshift) and of course provide ever precise laboratories for dynamical gravity ($v/c\approx 1$).

There will be continued value in the study of Galactic binaries both for detailed studies of the objects but also for population studies. After all, we do know the history of our Galaxy better than any other galaxy!

Stellar black holes have been primarily identified via X-ray astronomy: as high mass X-ray binaries (e.g. Cygnus X-1) and as low X-ray binaries (e.g. A0620−00). A schematic menagerie of Galactic stellar black holes can be found in Figure 1.2. Even though we know fewer than 50 black hole systems (to varying degrees of confidence) it is estimated that our Galaxy hosts 3×10^8 black holes (McClintock

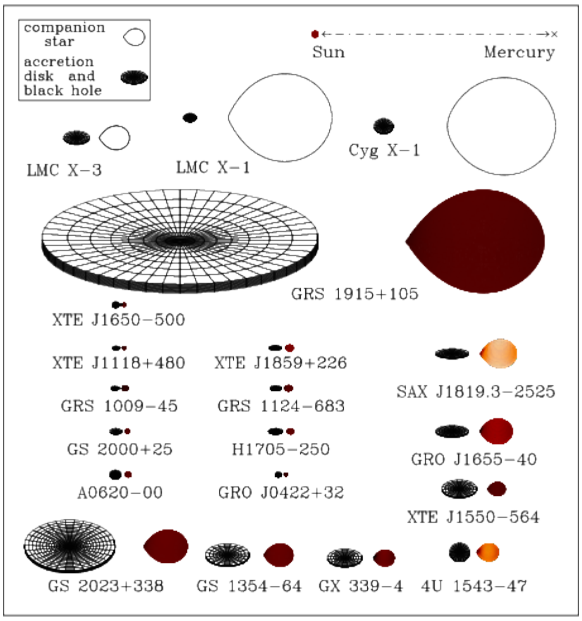

Figure 1.2: A schematic of 20 dynamically confirmed stellar black hole binaries to relative scale. From `https://jorosz.sdsu.edu/`, the homepage of Prof. Orosz at San Diego State University.

& Remillard 2006). Assigning an average mass of 10 $M_\odot$ this amounts to 4% of the total baryonic mass of the Galaxy.

The above discussion suggests an all out effort to find stellar black holes will be fruitful. These notes are focused on identifying methods by which Galactic stellar black holes, single or otherwise, can be found. In successive chapters we will discuss each of these methods for finding black holes. Each chapter starts with a discussion of the physics of the detection method and then summarizes the estimated yield of the method (from papers). Each chapter ends with discussion of a phenomenon or topic that is fundamental to black hole astrophysics.

BlackCAT–A resource: Corral-Santana et al. maintain a list of black holes and candidates at `http://www.astro.puc.cl/BlackCAT/` As of December 31, 2018 it had sixty entries. It is a very useful portal should you wish to undertake research on a sample of black holes.

Useful papers to read.

"X-ray properties of black-hole binaries" by R. A. Remillard & J. E. McClintock (Annual Reviews of Astronomy & Astrophysics 44, pp 49-92 (2006) is a comprehensive introduction to stellar black hole X-ray binaries.

Chapter 2

Overview & Landscape

Stars with $M < 8\,M_\odot$ end their lives as white dwarfs. There is sound evidence to this expectation in that white dwarfs are seen in young clusters with a turnoff mass up until $8\,M_\odot$. Low mass stars form low mass white dwarfs ($0.6\,M_\odot$) while stars close to $8\,M_\odot$ form $1.2\,M_\odot$. Much work has gone into deriving a mapping between the initial mass of the star and the final mass of the white dwarf. Astronomers have measured or robustly inferred the masses of white dwarfs. Excluding white dwarfs which are in binary systems (and thus have exotic origins) the masses range from $0.6\,M_\odot$ to $1.2\,M_\odot$.

Literature Search. *Gaia* has revolutionized white dwarf research. Peruse the literature to find the lowest mass of a single white dwarf and the most massive white dwarf.

It is clear that stars with mass greater than $8\,M_\odot$ form neutron stars but the dividing line between a neutron star outcome or black hole outcome is very much work in progress. Leaving aside this critical issue there are two big differences between the two outcomes. First, it is well established that neutron stars are imparted with velocity kicks. This natal velocity kick has profound effect on the survivability of binary neutron star system. It is not clear if the same is the case with black holes. Second, the masses of neutron stars is very tightly constrained, ranging from $1.2\,M_\odot$ to $2\,M_\odot$ with the highest mass corresponding to millisecond pulsars (which are expected to have accreted considerable amount of matter following the formation of the neutron stars). In contrast there is a bigger spread in black hole masses from $5\,M_\odot$ to $40\,M_\odot$ (see Figure 1.1).

2.1 Black Hole Masses

The masses of Galactic stellar black holes, all derived from optical observations of X-ray binaries, range from $5\,M_\odot$ to $15\,M_\odot$ (see Figure 1.2). It is significant that so far no black holes with mass less than $5\,M_\odot$ have been found. Massive black holes with mass reaching to $40\,M_\odot$ are seen in GW events occurring at high redshift (Figure **??**).

Unlike the case for white dwarfs but similar to that of neutron stars we most certainly lack an initial mass to final mass mapping relation for either neutron stars or black holes. With little doubt metalicity is expected to change the mass boundary between neutron star progenitor and black hole progenitor. However, there is increasing evidence that binarity may also influence the outcome (mass transfer, imparting angular momentum to the progenitor star). For simplicity, unless stated otherwise, we will assume that single stars with mass greater $20\,M_\odot$ will leave black hole residues.

A variety of such relations have been proposed: $M_{\rm BH} = kM$ where $k = 0.2$ (for $M > 20\,M_\odot$). This simple relation produced black holes ranging from $4\,M_\odot$ to $20\,M_\odot$. We will use the "curved" relation given in Yamaguchi et al. (2018; `ykb+18`):

$$\frac{M_{\rm BH}}{M_\odot} = \frac{2}{\log(3)}\log\Big(\frac{M}{M_\odot} - 19\Big) + 2 \qquad \text{for} M \geq 20\,M_\odot. \tag{2.1}$$

For $M = 20\,M_\odot$ we find $M_{\rm BH} = 2\,M_\odot$ and $M_{\rm BH} = 10\,M_\odot$ for $M = 100\,M_\odot$.

2.2 Initial Mass Function

The initial mass function (IMF) is the distribution of masses of stars at birth. It is stated that the IMF is "universal" and the stellar birth rate is reasonably approximated by a power law

$$\frac{dn}{dM} \propto M^{-\alpha} \tag{2.2}$$

where M is the mass of the stars as they are formed, dn/dM is the rate of newly formed stars with mass between M and $M = M + dM$. The star-formation rate is usually quoted as the integral of this quantity

$$\mathcal{S}(t) = \int \frac{dn}{dM} M dM \tag{2.3}$$

and the usual units are $M_\odot\,{\rm yr}^{-1}$. Spiral galaxies usually have very large star-formation when they are just being put together and then a steady rate whereas elliptical galaxies underwent rapid star-formation early on and with no current star-formation. For the past many billion years the star-formation rate in our Galaxy is about $2\,M_\odot\,{\rm yr}^{-1}$.

Edwin Salpeter was the first person to look at the data and propose the power law model and determined $\alpha = 2.3$.[1] The average mass of a star, $\langle M\rangle \approx (\alpha-1)/(\alpha-2)M_l$ where M_l is the lower mass cutoff for the power law and it is assumed that the upper cutoff, $M_u \gg M_l$. Many times you will find $M_l = 0.1\,M_\odot$ but there is little basis to this. In the solar neighborhood star-formation rate peaks at $\approx 0.3\,M_\odot$ (CHECK). For $\alpha > 2$ M_u does not matter much. For the Salpeter value, $\langle M\rangle \approx 4.3M_l$. The fraction of new stars with $M > 20\,M_\odot$ is 0.0022 whereas the mass fraction in black hole progenitors is 11%.

[1] There are variants for defining α such as $\alpha + 1$ etc. Beware.

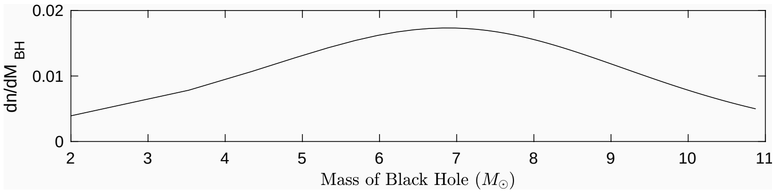

Figure 2.1: The probability distribution of black masses, $dn/dM_{\rm BH}$, assuming a power law IMF, $dn/dM \propto M^{-\alpha}$ with $\alpha = 2.3$ over the mass range, $20 < M < 150\,M_\odot$ and a "curved" initial-mass to final-mass relation of Yamaguchi et al. (2018).

Problem P1 With reference to Figure 2.1 explain why the distribution of $M_{\rm BH}$ is strongly peaked.

Problem P2: Number of Black Hole Binaries. Assume that the star-formation rate in our Galaxy has been constant for the past 10 Gyr and equal to $\mathcal{S} = 2\,M_\odot\,{\rm yr}^{-1}$, the IMF is a Salpeter power law between $0.2\,M_\odot$ and, say, $100\,M_\odot$ and that all stars with $M > M_* = 20\,M_\odot$ end their life as black holes. Compute the number of black holes produced to date.

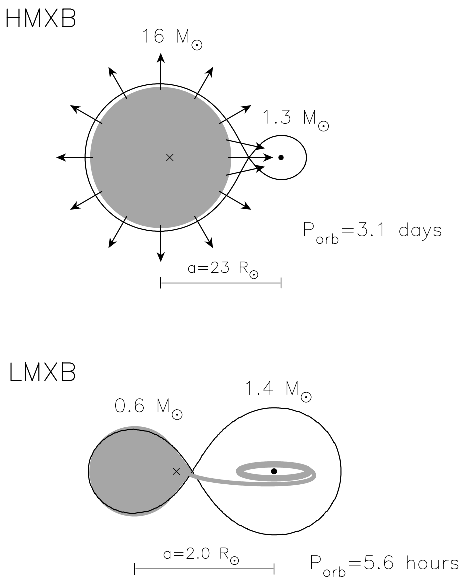

Figure 2.2: High mass X-ray binaries (HMXB) in which X-rays are generated by a neutron star accreting matter from stellar wind of a massive star and low mass x-ray binary (LMXB) in which X-rays are generated by a neutron star accreting from a companion losing matter via its Roche lobe (from Tauris & van den Heuvel 2006 [th06]). The same architecture holds if we replace neutron stars by black holes. We will use the words LMXBs and Low Mass Black Holes (LMBH) to mean a binary containing a black hole with the low mass referring to the mass of the companion, say $0.5\,M_\odot$ to a few $M_\odot$. Likewise HMXBs and High Mass Black holes (HMBH)refer to binaries with black holes and companions which are massive, $\gtrsim 10\,M_\odot$.

2.3 Galactic Blackholes: Tip of the Iceberg

An isolated black hole in a vacuum does not emit light (well, almost). It can be sensed via its gravitational and a possible approach is via micro-lensing of Galactic stars which is not an easy experiment to undertake. However, black holes have deep potential well and so accreting black holes are quite luminous. Much of our knowledge of Galactic binaries comes from black holes in binary stars which accrete matter from stellar wind of the companion (high mass binaries; HMBH) or accretion via Roche lobe overflow (low mass binaries; LMBH); see Figure 2.2. The prototypes are, respectively, Cyg X-1 and A0620−00. The two classes are summarized in Chapters 5 and 6.

The closest black hole to us is an LMBH, A0620−00, located at a distance of about 1 kpc. As noted towards the end of Chapter 1, BlackCAT has sixty entries, as of December 31, 2018. Of these only five (including LMC, M33) are HMBH and the others are black hole binaries with lower mass companions.

Exercise E1: Closest Black Hole. The local white dwarf density estimated from *Gaia* is $(4.49 \pm 0.4) \times 10^{-3}\,\mathrm{pc}^{-3}$ (Hollands et al. 2018; `[htg+18]`). Scaling from this and using the assumptions of the previous problem estimate the local black hole density. What is the median distance to the nearest black hole, given this density. Compare this distance to that of the nearest known black hole, A0620−20. [See Kulkarni & van Kerkwijk 2010 `[kvk10]` for pitfalls of deductions based on nearest object.]

Exercise E2: Velocity Kicks. How would you answer change if black holes were born with a velocity dispersion of, say, $50\,\mathrm{km\,s^{-1}}$ or $100\,\mathrm{km\,s^{-1}}$? For inspiration investigate the literature about the local neutron star density and the nearest neutron star.

The formation of these binary systems is quite delicate and involves mass transfer, order of magnitude shrinkage in the orbital radius and both steady and episodic mass loss. In fact, some of this can be deduced from the most elementary consideration. From Kepler's equation (Equation 3.7) we see the orbital radius is $a = 1.1(M/10\,M_\odot)^{1/3} P_{\rm hr}^{2/3}\,R_\odot$ where $P_{\rm hr}$ is the orbital period in hours and $M = M_{\rm BH} + M_2$ with M_2, the mass of the companion. The orbital separation ranges from $3\,R_\odot$ (V518 Per or XTE 0422+32; $P = 5.1\,$hr) to $95\,R_\odot$ (GRS 1915+105; $P = 800\,$hr). The mass of the progenitors of black holes range is $20\,M_\odot$ and up. Thus, the progenitor of the black hole in XTE 0422+32 would not fit in the current orbit. Thus the initial binary must have been large enough to accommodate the progenitor star of the black hole. However, following the formation of the black hole the orbit must have shrunk to the present value. This critical step is called as the "common envelope" phase and is now a vigorous observational and theoretical area of inquiry.

Let us review our current idea of the formation of the A0620−00, the prototype for LMBH (Figure 2.3). The progenitor of the black hole is suitably massive star, $\approx 40\,M_\odot$. The orbital radius is about $900\,R_\odot$ ($P_b = 500\,$d) which allows for the progenitor star to fully evolve and form a dense core. Massive stars have strong stellar winds and lose matter right from the birth. The gradual mass expands the orbit ($P_b = 750\,\mathrm{d}$). On ascending the giant branch the progenitor fills its Roche lobe and mass transfer ensues. When mass is transferred from a more massive star to a less massive star (in this case, the secondary is a $1\,M_\odot$ star) the orbit shrinks. The result is an unstable mass transfer aka common envelope evolution and the end product is a binary with only the Helium core of the primary star. The process happens so quickly that the secondary has no time to accrete significant matter. The helium core evolves and has very strong stellar winds. It eventually explodes and leave a black hole. If the orbital period of the binary is now short enough, $< 20\,\mathrm{hr}$ then GR loss is large enough to bring the secondary into Roche lobe contact. The resulting mass transfer makes the binary an X-ray source and that is how A0620−20 becomes detectable.

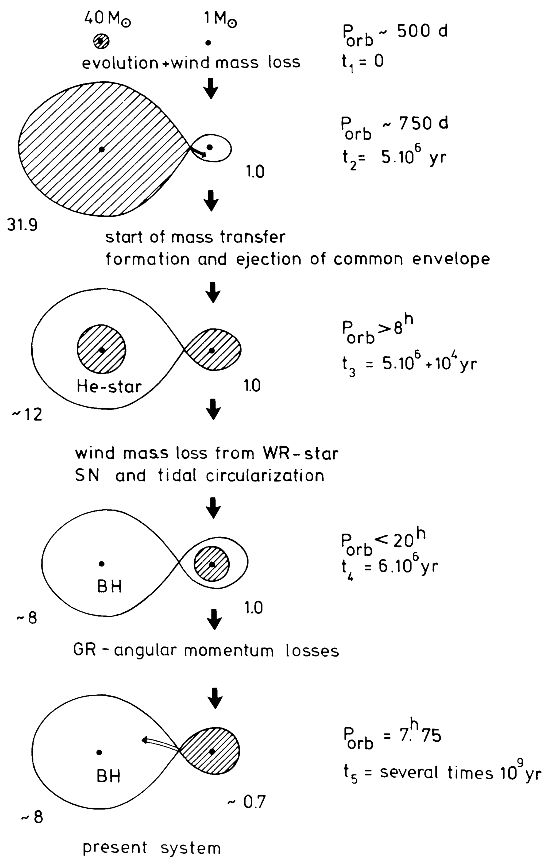

Figure 2.3: Formation scenario for A0620−20. From de Kool, van den Heuvel & Pylyser (1987; [khp87]).

The scenario discussed above clearly does not work if the initial binary was very wide or very compact. So for every A0620−20 we see there are many binary progenitors in which the black hole does not accrete matter (wide system) or the two stars merge (compact systems). Clearly, the observed Black Hole X-ray binaries represent a small fraction of Galactic black holes.

2.4 Population Studies: An Introduction

Population studies are motivated to construct the underlying population of interesting objects. Population studies start off with some sort of initial mass function (IMF) for the primary star (M_1), the binarity fraction and the distribution functions for: $q = M_2/M_1$ for the secondary star (by definition, $M_2 < M_1$), initial orbital separation (a_0) and eccentricity of the initial orbit (e_0).

The primary (the more massive star) evolves and increases in radius. It is possible to then have mass transfer which is either stable or unstable, depending on the value of q and also a_0 (in the sense of the structure of the primary star when it reaches the Roche lobe). An additional complication is that massive stars undergo both steady and episodic mass loss during the entire course of their life. These losses will expand the initial orbit. Finally, like neutron stars (for which the phenomenon is well established) it is possible that black holes may receive a velocity "kick" at birth. If so, wide binaries especially those with low mass companions will be disrupted. Clearly binary evolution is considerably more complicated than the evolution of a single star (which itself is frontier field when it comes to massive stars). It is for these reasons that we have an entire cottage industry devoted to binary population modeling.

Recently there has been a flurry of studies to estimate the number of Galactic black holes that will be detected via the astrometric method, specifically the *Gaia* mission. I recommend Kinugawa & Yamaguchi (2018; `[ky18]`) for a clear exposition of the elements that are needed for population studies. We will make use of these models in the next two chapters.

Exercise E3: Initial orbital radius. Population studies usually assume that the initial binary separation is "logarithmically flat". This means the number of binaries with orbital radius between a_0 and $a_0 + da_0$ is $dn = (A/a_0)da_0$ where A is the normalization set by the requirement

$$\int_{a_1}^{a_2} \frac{dn}{da_0} = 1. \tag{2.4}$$

What are reasonable values for a_1 and a_2? [Difficult: how would your answer change if you allow for eccentric orbits?]

Chapter 3

Astrometry

Astrometry is the most classical method by which masses are traditionally measured in astronomy. In, fact if you can measure the orbit of each of the components in a binary then all the key parameter are measured. To date, no stellar black hole has been discovered by this technique. However, estimates suggest that the *Gaia* mission will identify a few hundred Galactic black hole binaries (BHBs). Furthermore, it is via astrometry that the mass of the nuclear black hole of our own Galaxy has been securely measured.

3.1 Two Body Problem

Consider two bodies, mass M_1 and M_2 located at $\mathbf{r_1}$ and $\mathbf{r_2}$. Let $\mathbf{a} = \mathbf{r_2} - \mathbf{r_1}$. The gravitational force felt by body 1 due to body 2 is

$$\mathbf{F}_{12} = -\frac{GM_1M_2}{a^3}\mathbf{a} \tag{3.1}$$

and the corresponding acceleration is

$$\frac{d^2\mathbf{r_1}}{dt^2} = \frac{GM_2}{a^3}\mathbf{a}. \tag{3.2}$$

Symmetrically, the force felt by body 2 due to body 1 is $F_{21} = -F_{12}$. The sum of the two forces is zero and thus we have

$$M_1\frac{d^2\mathbf{r_1}}{dt^2} + M_2\frac{d^2\mathbf{r_2}}{dt^2} = 0. \tag{3.3}$$

The evolution of the "center-of-mass" (CM) is given by integrating Equation 3.3. With no loss of generality, we will set the integration constant to zero:

$$M_1\mathbf{v_1} + M_2\mathbf{v_2} = 0. \tag{3.4}$$

Here, $v_k = d\mathbf{r_k}/dt$ with $k = 1, 2$ is the velocity of particle k. Integrating, Equation 3.4 yields the coordinate of the CM:

$$\mathbf{R} = \frac{M_1\mathbf{r_1} + M_2\mathbf{r_2}}{M}. \tag{3.5}$$

Here, $M = M_1 + M_2$ and our choice of the normalization ensures that the CM lies on the vector connecting the two masses.

Problem P1. Show that the CM lies on the vector connecting M_1 to M_2. Why is it desirable for the CM to be defined so that it lies on $\mathbf{a}$?

Next, the difference in the accelerations is

$$\frac{d^2\mathbf{a}}{dt^2} = -\frac{GM}{a^3}\mathbf{a} \tag{3.6}$$

The straightforward interpretation of this equation is that we have a unit mass ("test") particle that feels the combined gravitational attraction of the two bodies. Let us assume that the resulting orbit of this test particle is a circle with an orbital period, $P = 2\pi/\omega$. In the rotating frame of the orbit the test particle is at rest. Thus, we conclude that the attractive force is balanced by the centrifugal force:

$$\frac{GM}{a^2} = \frac{v^2}{a} = \omega^2 a \tag{3.7}$$

where $v = a\omega$ is the tangential velocity of the particle. This leads to the famous third law of Kepler:

$$GM = a^3\omega^2. \tag{3.8}$$

Solution to Equation 3.6 exist and are extensively discussed in elementary books on Newtonian mechanics.

Most of us have been taught of reduced mass and Equation 3.6 is frequently written with reduced mass on the LHS instead of test particle. Let us understand the motivation for reduced mass. To this end, we compute the orbital angular momentum: $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ where $\mathbf{p}$ is the linear momentum. The orbital angular momentum has two contributions:

$$L = M_1 a_1 v_1 + M_2 a_2 v_2 \tag{3.9}$$

where a_k and v_k are the orbital separation from the CM and orbital velocity with respect to the CM, respectively. From Equations 3.4 we have $M_1 v_1 = M_2 v_2$ and from 3.5 we have $M_1 a_1 = M_2 a_2$. It is easy to show that

$$a_1 = \frac{M_2}{M}a = \frac{\mu}{M_1}a \tag{3.10}$$

where

$$\mu = \frac{M_1 M_2}{M} \tag{3.11}$$

is the so-called reduced mass. With this relation in hand and noting that $v_k = a_k\omega$ we have

$$L = \mu a^2 \omega. \tag{3.12}$$

This equation implies that the orbital angular momentum is given by a single particle with mass μ and tangential velocity, $v = a\omega$. This discussion motivates the concept of reduced mass. For this purpose, multiply μ on both sides of Equation 3.6 and you will find

$$\mu\frac{d^2\mathbf{a}}{dt^2} = -\frac{GM_1M_2}{a^3}\mathbf{a}. \tag{3.13}$$

The physical interpretation of Equation 3.13 is a particle with mass, μ, is acted upon by an attractive force equal in magnitude to F_{12} or F_{21}. As with angular momentum, the kinetic energy and the potential energy of the reduced mass correspond to the kinetic energy and potential energy of the binary system.

3.2 *Gaia*

The angular size of the orbit is a/d where d is the distance to the binary. From Equation 3.7 we can compute a, given M_1, M_2 and orbital period, P. However, it helps to work in "natural" units in which most quantities are close to unity. To this end, let $m_k = M_k/M_\odot$. Next, we realize by application of Kepler's third law to the Sun-Earth system that

$$m = \left(\frac{a}{\mathrm{AU}}\right)^3 \left(\frac{\mathrm{yr}}{P}\right)^2 \tag{3.14}$$

where $m = m_1 + m_2$. Furthermore, by the definition of parsec, one AU subtends an angle of 1 arcscond at a distance of 1 parsec. Thus, the orbit subtends a radius of

$$\theta = m^{1/3}\left(\frac{d}{\mathrm{kpc}}\right)^{-1}\left(\frac{P}{\mathrm{yr}}\right)^{2/3} \mathrm{mas} \tag{3.15}$$

where mas stands for milli-arcsecond. Note that unlike other techniques, in the case of black-hole binaries, astrometric detection leads to unambiguous determination of the mass of the black hole (though limited in precision of the mass estimate by the astrometric precision, including that of the parallax).

Going forward, we will assign index 1 to the black hole and index 2 to the companion star. The radius of the orbit of the companion star is

$$\theta_2 = \frac{M_1}{M_1 + M_2}a = \frac{1}{1+q}a \tag{3.16}$$

where $q = M_2/M_1$. Thus detection of a black hole via astrometry requires milli-arcsecond precision, given a typical distance of 1 kpc (or more). Fortunately, Gaia mission is capable of this meeting this requirement (see Table 3.1).

Spectral Type	B1V	G2V	M6V
V=15 mag	26 μas	24 μas	9 μas
V=20 mag	600 μas	540 μas	130 μas

Table 3.1: Parallax error at the end of 5-year Gaia mission

There have been a flurry of papers which have undertaken population studies with the goal of predicting the number of astrometric black hole binaries that Gaia will find I recommend Yamaguchi et al. (2018; [ykb+18]) and Yalinewich et al. (2018; [ybh+18]).

Literature Search. Read up on the *Gaia* mission and be ready to summarize the key performance shown in Table 3.1.

Exercise 1. Assemble a table of optical counterparts to Galactic black hole binaries. Cutoff the list at $V > 20$ mag. Compute the angular radius of the orbit of the secondary star in each case.

Exercise E2: Minimum & Maximum Mass of Black Holes. What is the significance of the limiting masses for black holes? For instance, would you be surprised if a black hole of mass $2\,M_\odot$ was discovered? How about a black hole of mass $100\,M_\odot$ orbiting, say, a $20\,M_\odot$ star?

Exercise E3: Precision Astrometry. All high precision techniques (e.g. astrometry via imaging as in *Gaia* or phase shift in spatial interferometers) eventually boil down to some sort of point-spread function fitting. The precision is usually a tiny fraction of the point spread. With reference to Figure 3.1 show that the precision with which you can determine the location of the point spread function is

$$\sigma_x = \frac{\mathrm{FWHM}}{\mathrm{SNR}} \tag{3.17}$$

where FWHM is the full width at half-maximum of the point spread function and SNR is the signal-to-noise ratio. Specifically, for the assumptions made here (Figure 3.1) we have $\sigma_x = \sigma/(A\sqrt{2})$ where σ is the noise in each pixel (assumed to be independent of the number of photoelectrons) and $2A$ is the total signal strength.[1]

This homework should make you appreciate how *Gaia* with a $1.45\,\mathrm{m}\times0.5\,\mathrm{m}$ primary mirror (which corresponds to a Rayleigh criterion angular resolution of $69\,\mathrm{mas}\times200\,\mathrm{mas}$) can achieve ten microarcsecond precision.

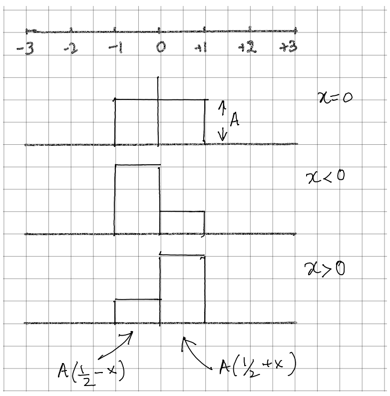

Figure 3.1: A schematic of a two pixel detector. The two pixels are located between 0 and +1 (hereafter "R") and 0 and −1 (top panel). We will assume that the point spread function is a "top hat" function with a width of one pixel (FWHM=1). In the next panel (moving down) the signal is centered at x=0 and so each pixel gets the same number of photoelectrons: A. In subsequent panels (moving downwards) the signal is centered at $-1/2 < x < 0$ and $1/2 > x > 0$, respectively. Consider the bottom most panel. In this case, the number of photoelectrons in the R pixel is $A(1/2 + x)$ and $A(1/2 - x)$ in the L pixel.

[1] We have ignored Poisson fluctuations. The correct answer is $\sigma_x = (\sigma^2/4 + A/2)^{1/2}A^{-1}$.

Chapter 4

Radial Velocity

It is only this year (2018) that the first stellar mass black hole binary was discovered using the methodology of radial velocity (RV). In contrast, all extra-galactic black holes, stellar or nuclear, have been identified by the RV methodology. The power of RV method is that, unlike that for astrometry, the primary signal sensitivity does not decrease with distance. This explains why RV is used for extra-galactic sources. However, RV is limited by the fact that it can only yield lower limit to the mass of the black hole.

The radial velocity of the companion is $v_2 = a_2\omega$ and can, in most instances, be measured via optical or NIR spectroscopy. Rewriting Equation 3.7 we find that the total velocity amplitude is

$$v = 458\Big(\frac{M_1}{10\,M_\odot}\Big)^{1/3}(1+q)^{1/3}\Big(\frac{P}{\mathrm{day}}\Big)^{-1/3}\,\mathrm{km\,s^{-1}}. \tag{4.1}$$

Let us assume that the orbit is circular and let i be the angle between the orbital angular momentum and the line of sight. Thus $i = 0$ means that the orbit is in the plane of the sky whereas $i = 90°$ means that the orbit is perpendicular to the plane of the sky. The radial velocity of the secondary star is $v_2 \sin(i)$ where

$$v_2 = \frac{M_1}{M}v = 458\Big(\frac{M_1}{10\,M_\odot}\Big)\frac{1}{(1+q)^{2/3}}\Big(\frac{P}{\mathrm{day}}\Big)^{-1}\,\mathrm{km\,s^{-1}}. \tag{4.2}$$

Thus for binaries with orbital periods of fraction of days to months the velocity is within reach of massively multiplexed surveys such as LAMOST, DESI and SDSSV (and is trivial for MKE). The time for RV methodology to shine has finally arrived!

4.1 The Binary Mass Function

Eliminating a between $v_2 = (1+q)^{-1}a\omega$ and Equation 3.7 (Kepler's law) we find

$$M_1^3 = \frac{K^3M^2}{G\omega} \tag{4.3}$$

where K is now the semi-amplitude of v_2. This Equation can be recast into a function of observables (LHS) and model parameters (RHS):

$$f = \frac{PK^3}{2\pi G} = \frac{M_1^3 \sin(i)^3}{M^2}. \tag{4.4}$$

The quantity f derived from observations is the so-called "binary mass function". As can be seen from the RHS f has the unit of mass. The mass function, in absence of any additional knowledge of i or the mass of the secondary, yields only the minimum mass for the black hole. Knowledge of f is somewhat useful in two asymptotic regimes: (1) when the other object is an exoplanet, $M_1 = f^{1/3} M_2^{2/3}/\sin(i)$ where M_2 the mass of the parent star which is well known and (2) when the other object is a massive black hole ($M_1 \gg M_2$) in which case $M_1 = f/\sin^3(i)$. Usually for XRBs, the mass of the companion, M_2, can be estimated using optical or NIR spectroscopy. Then one can solve Equation 4.4 as a function of i and infer a probability distribution for M_1. From the above discussion it should be clear why astronomers highly value eclipsing binaries ($i \approx 90°$).

Problem P1. Show that f is the minimum possible mass for the black hole.

A0620−00. The X-ray nova, A0620−00 is the prototype of low mass black binary. It is perhaps one of the most studied black hole binaries. The orbital period is 0.323 days and the companion is a late type (K4 or so) star. González Hernández & Casares (2010; [gc10]) undertook high spectral resolution spectroscopy of the source in quiescence (when the visible light is dominated by the companion) and find $K_2 = 437 \pm 2\,\mathrm{km\,s^{-1}}$. They further find the companion is rotating rapidly (line broadening, consistent with a high level of chromospheric activity). The companion is filling its Roche lobe and thus can expected to be spin-locked to the orbit. Thus, $q \equiv M_2/M_1 = 0.062 \pm 0.01$. From Equation 4.4 we see that $M_1 \sin(i) = f(1+q^2)$. Thus, M_1 must be at least $2.79 \pm 0.04\,M_\odot$ ($q = 0$) and greater than $3.15 \pm 0.10\,M_\odot$ adopting the above value of q.

The $\sin(i)^3$ is severe enough that mass estimates obtained from the mass function is basically good to set lower limits but not more than that. The companion is filling its Roche lobe and thus it will be distended (in the direction towards the black hole). The resulting "ellipsoidal" modulation (discussed later in the notes) gives us an opportunity to measure i. There are unfortunate complications (contribution from the disk, chromospheric activity and so variability). Estimates for i range from 51° to 58° (the uncertainty is due to contribution to the photometry from the disk) which leads to a black hole estimate of about $7\,M_\odot$ (van Grunsven et al. 2017; [gjv+17]).

Exercise E1. In order to appreciate the dependence on i plot the cumulative probability distribution of M_1 for A0620-00 given P, K and q.

4.2 The First Detection of a Black Hole by RV

MUSE (Multi-Unit Spectroscopic Explorer) is a state-of-the art spectrograph with 24 integral field unit (IFU) optical spectrographs, each with $1' \times 1'$ entrance field-of-view ($0.2'' \times 0.2''$ spaxels). This formidable instrument has been undertaking RV studies of Galactic globular clusters. Giesers et al. (2018; [gdh+18]) report a star at the main sequence turnoff, in the globular cluster NGC 3201 (distance, 4.9 kpc) which exhibited strong radial velocity variations (Figure 4.1). The "target star" (i.e. the companion) is $0.8\,M_\odot$ just above the main sequence turnoff. The authors report a period, $P = 166.9$ day, a semi-amplitude, $K = 69.4 \pm 2.5\,\mathrm{km\,s^{-1}}$, and eccentricity, $e = 0.595 \pm 0.022$.

For an eccentric binary the mass function is

$$f = \frac{M_1^3 \sin^3(i)}{M^2} = \frac{K_2^3 P}{2\pi G}(1-e^2)^{3/2}. \tag{4.5}$$

For the parameters above, $f = 3\,M_\odot$. However, the value of M_2 is known we can get a stronger lower limit on the mass of the black hole, $M_1 > 4.24\,M_\odot$.

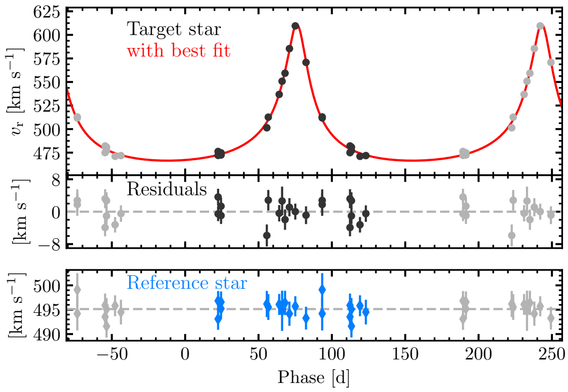

Figure 4.1: Radial velocity light curve of the black hole binary system in NGC 3201. From Giesers et al. (2018; [gdh+18]).

4.3 A Search for BH binaries with Highly Multiplexed Spectrographs

We are in growth phase of highly multiplexed spectrographs (see Figure 4.2). While the first generation spectrographs boasted hundreds of spectral channels LAMOST kicked off the era of highly multiplexed the second generation has thousands of fibers (e.g. LAMOST, DESI, PFS etc). This opens up the possibility of searches for companions via large scale radial velocity surveys.

We first develop a toy model. In this model, neutron stars are the end products of main sequence stars with mass $8\,M_\odot < M < M_*$ whereas black holes emerge from stars with $M > M_*$. We set $M_* = 20\,M_\odot$. Given the age of the Universe we will only restrict to $M > 0.8\,M_\odot$ for significant stellar evolution. We will assume a Salpeter IMF with lower cutoff of $0.2\,M_\odot$ and upper cutoff of $100\,M_\odot$. For even more simplicity we will assume that all $> 8\,M_\odot$ stars are born as binaries. However, a good fraction of neutron stars will not survive the formation of the neutron star (natal velocity kicks). We are making the implicit assumption that black hole binaries may survive the formation of black holes.

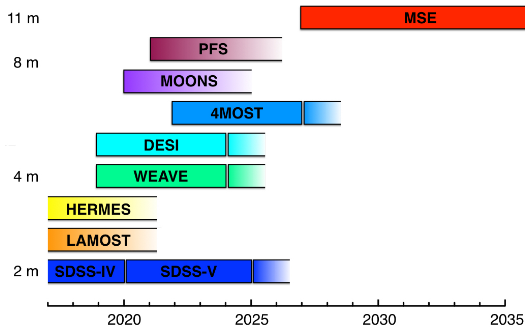

Figure 4.2: The landscape of large field-of-view multi-object spectrographs. From the Maunakea Explorer (MSE) book.

Our goal is to undertake large scale radial velocity surveys with repeated visits. We need to estimate the cadence and number of visits. Let us, for now, set it 10 visits over 3 years. To this end, we would ideally like to ideally target binaries which are likely to host neutron stars or black holes. With the above assumptions, the fraction of white dwarfs, neutron stars and black holes is 15.7%, 0.58% and 0.22%, respectively. Taken at face value, less than 1% of stars will be hosting either a neutron star or a black hole. Thus in order to unearth a neutron star or black hole we would need 10×100 repeat observations. This requirement is steep and so we need to refine our approach.

O and B stars lack strong lines. Since our goal is to select binaries based on the radial velocity method we will restrict to those binaries in which the brighter star (hereafter, the photo-primary) is later than spectral type B (i.e. A–M spectral types). The second star, by construction, is fainter than the photo-primary and can be a (1) main sequence star or (2) a low mass companion (sub-stellar), (3) a white dwarf or (4) or no star at all or (4) a neutron star or black hole degenerate star. Our goal is to get rid of binaries with possibilities (1), (2) and (3). Unfortunately, possibilities (3) and (4)) may have significant overlap with (5) and arguably even (2) overlaps with (5).

The tools we have at our disposal are (1) precision multi-band PS1 photometry, (2) the mid-infrared (MIR) photometry from WISE, (3) NUV flux from *Galex* and (4) parallax from *Gaia*. Our plan of attack is as follows: use a set of simulated binaries from Kinugawa & Yamaguchi (2018; [ky18]) and subject them to these observational constraints. Estimate the fraction of candidates that remain. Then estimate the amount of time on DESI that is needed to identify systems with orbital period of less than say 3 years.

Primer: Mass Function for Eccentric Orbits

Chapter 5

X-ray Binaries: Massive Companions

Sco X-1 was the first extra-solar X-ray source and discovered in 1962 by a sounding rocket flight designed to detect possible X-ray emission from the moon. In 1964 Cygnus X-1 was discovered by another sounding rocket. Radio emission was detected in 1971 and the resulting arc-second localization showed a bright star, HD 226868. Rapidly thereafter optical astronomers showed that the mass function derived from radial velocity observations of the O companion (mass, $15\,M_\odot$) was $0.244\,M_\odot$ and thereby a minimum mass for the companion in excess of $3\,M_\odot$. LMC X-3, a bright X-ray source in the Large Magellanic Cloud (LMC), was similarly shown to be a black hole binary in 1983. This source, like Cygnus X-1, has a massive companion. This group is now called as Black Hole High Mass X-ray binaries (HMXBs) where the term "high mass" refers to the mass of the companion to the black hole.

It is surprising (to me, anyway) that there are only five such systems in the Local Group (let alone our own Galaxy; see Table 5.1 which is copied from van den Heuvel (2018; in prep). Naively, the progenitor masses of the black holes must be higher than that of the companion. Thus, some of these black holes can be residues of truly massive stars, $\gtrsim 70\,M_\odot$. Next, notice the extreme mass loss between the mass of the progenitor star and the black hole mass.

Table 5.1: Known Black Hole HMXBs

Name	P(d)	$M_2(M_\odot)$	$M_{\rm BH}(M_\odot)$
Cyg X-1	5.6	19.2 ± 1.9	14.8 ± 1
LMC X-1	3.9	31.8 ± 3.5	10.9 ± 1.4
LMC X-3	1.7	3.6 ± 0.6	7 ± 0.6
MCW 656	~ 60	~ 13	4.7 ± 0.94
M33 X-7	3.45	70 ± 7	15.6 ± 1.5

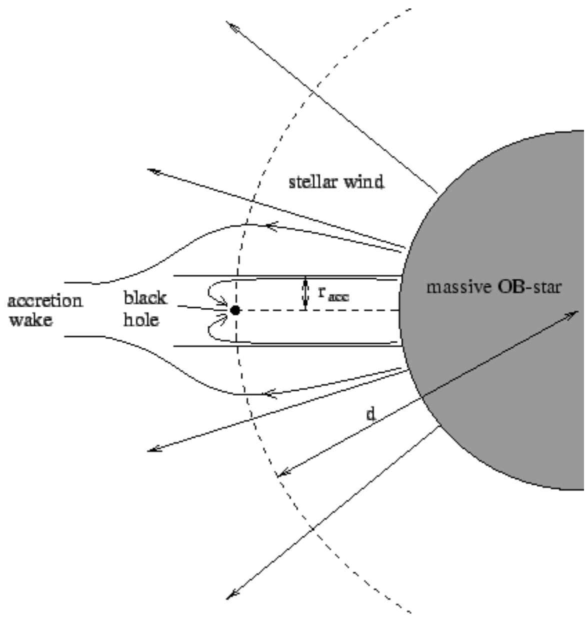

Figure 5.1: From `https://ttt.astro.su.se/groups/head/sara/node5.html`

Problem P1: Conservative Mass Transfer. Consider a binary star system with two stars, mass M_1 (primary) and M_2 (secondary; $M_1 > M_2$) orbiting each other in a circular orbit. Assume that mass is transferred from the primary star to the secondary star but with no loss of angular momentum. Show that the orbit contracts until the two masses are equal and thereafter, should mass transfer continue, the orbit will then expand. Try to understand this result in an intuitive way.

Problem P2: Unbinding a binary. Say the primary star, explodes and immediately loses ΔM. Show that the binary will unbind if $\Delta M > M/2$ where $M = M_1 + M_2$. Does the outcome depend on the eccentricity?

Chapter 6

X-ray Binaries: Low Mass Companions (X-ray Novae)

The X-ray sky is filled with a zoo of transient sources. The transient X-ray signal arises from a compact star (black hole or neutron star) accreting matter from a donor star (usually a hydrogen burning star). The transients can be further segregated by the mass of the donor star: low mass ($\lesssim 1\,M_\odot$), high mass ($\gtrsim 7\,M_\odot$) and intermediate mass. Here, we focus on a class variously called "X-ray novae" or "Ultra Soft X-ray Transients" (as opposed to hard and soft transients which are associated with magnetized and unmagnetized neutron stars).

The prototype for this class of sources is the A 0620−00. It was discovered as a strong X-ray transient in 1975 by the Ariel mission. It took 10 years before astronomers got around to undertaking detailed studies of the counterparts. Their efforts were rewarded when the mass function obtained from spectroscopic observations of the K spectral type companion yielded a mass function, $f = 3.18 \pm 0.16\,M_\odot$.

An example light curve[1] of an X-ray nova is shown in Figure 6.1. As can be seen from this light curve the sources are bright and long lived (hundreds of days), thereby affording followup.

Project: Recurrent Nova. A0620−00 is a recurrent nova, having undergone a nova explosion in 1917 (and detected in the optical). See if it makes sense to undertake historical searches, as with DASCH[2], for other X-ray novae (including MAXI discovered novae).

[1]This is a good occasion to learn the archaic but useful unit of X-ray astronomy called the Crab. As implied by the name it is the flux density of the Crab nebula observed by an X-ray photometer. In the "classical" X-ray band of 2–10 keV one Crab equals $2.4 \times 10^{-8}\,\mathrm{erg\,cm^{-2}\,s^{-1}}$. Since the Crab nebula has a spectral flux density which can be described by a power-law, $f_\nu \propto \nu^\alpha$ with $\alpha = -1$ (or Photon-index, $\Gamma = 2$), one Crab corresponds to approximately, 1 mJy!, at mid-band ($\approx 6\,\mathrm{keV}$).

[2]`http://dasch.rc.fas.harvard.edu/project.php`

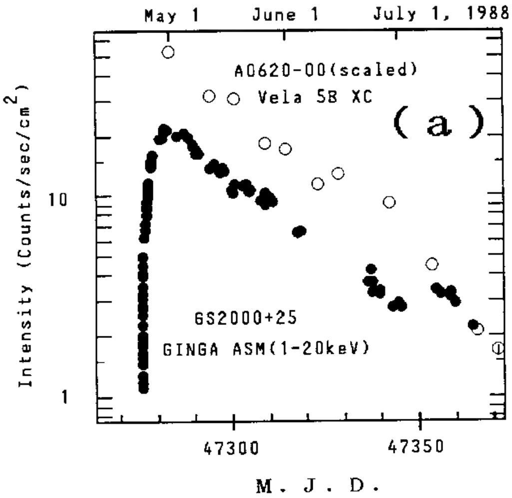

Figure 6.1: The light curve (solid circles) of GS 2000+25, an X-ray nova detected by the All-Sky Monitor aboard the Japanese Ginga (Astro-C) X-ray mission ([tko+89]; Tsunemi et al. 1988). The peak intensity is 12 Crab (1–6 keV). The light curve of A0620−20 are shown by open circles. GS 2000+25 arose in a binary system consisting of a black hole (mass, $\approx 7\,M_\odot$) and a late K-type companion ($0.5\,M_\odot$) with an orbital period of 8.26 hr. The system is located in Vulpecula constellation at a distance of about 2.7 kpc.

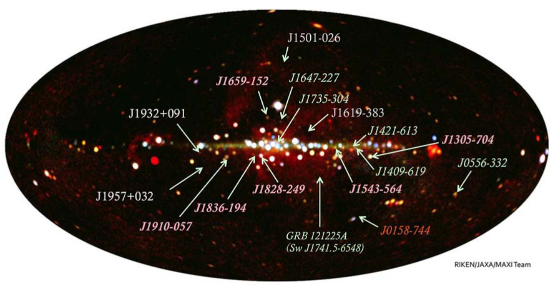

Figure 6.2: Seventeen MAXI transients found during the first 7 years of operation of the Japanese MAXI mission ([N17]; Negoro 2017). The designations of the six black hole candidates are shown in bold italic whereas the six neutron star transients in green and the sole white dwarf transient in red. The four unclassified transients are shown in vertical Roman font. From Negoro 2017 [N17].

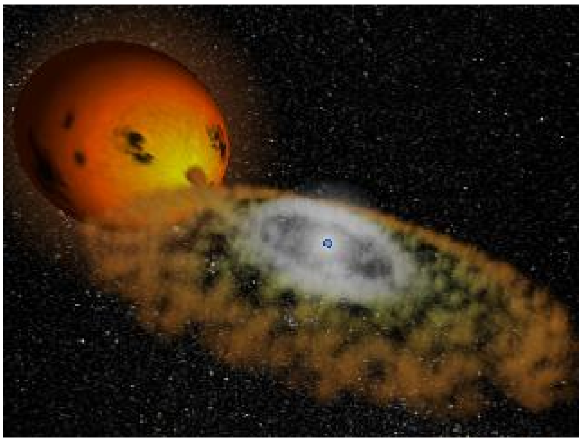

Figure 6.3: Illustration of dwarf nova.