νf_ν

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Abstract

Spectroscopists routinely use I_ν or I_λ to plot the spectrum; here, I_ν is the spectral flux density and ν is the frequency and $\lambda = c/\nu$. Astronomers use $\mathcal{F} = \nu I_\nu$, especially when plotting the spectra of powerful sources and the cosmic background; here, I assumed that the concept of $\mathcal{F}$ was in vogue in historical times (as in the the era of Planck and Wien). Apparently, not. In this pedagogical note I trace the history of the advent of νI_ν (and the credit goes to a radio astronomer and an IR astronomer). Next, it is commonly assume that νI_ν is an excellent surrogate for the bolometric output (that is the spectrum integrated over frequency). I examine this expectation by reviewing two models: black body emission and power law. I present a small new result for the latter. Finally, I caution against the use of νI_ν when line emission is involved (e.g., as in gamma-ray astronomy).

1 Spectral Flux Density

Let F be the flux density (energy per unit time per unit area). The spectral density is the energy received per unit time in a specific bandwidth is $f_\nu = dF/d\nu$. Radio astronomers prefer frequency spectral flux density with the bandwidth set to 1 Hz, $dF/d\nu$. Optical and IR astronomers prefer to use wavelength rather than frequency and so use $f_\lambda = dF/d\lambda$. Conservation of energy requires $f_\nu d\nu = f_\lambda |d\lambda|$. This relation allows you to convert f_ν to f_λ and vice-versa. Incidentally, this equality shows that

$$\nu f_\nu = \lambda f_\lambda.$$

2 The use of $\nu f(\nu)$

Wilhelm Wein discovered his eponymous "displacement" law in 1893. It was based on the peak of f_λ. It took 60 years before Bracewell [1] pointed out that the peak frequency (or wavelength) depends on the quantity being maximized, f_λ or f_ν. He argued that the ideal quantity to consider is

$$\mathcal{F}(\nu) \equiv \nu f(\nu) = \lambda f(\lambda) \equiv \mathcal{F}(\lambda) \tag{1}$$

with the virtue that F_λ (or F_ν) being independent of the unit of the spectrum (ν or λ). The primary point in Bracewell's paper is determining the energy of the "typical" photon (important for biological systems like our own eye). For $T_\odot = 5,800\,$K, the wavelength of the photon at the peak of νf_ν is 6117 Å which is sensible as opposed to 4938 Å at the peak of f_λ and 8156 Å at the peak of f_ν. Going forward, let $\nu'_{\rm p}$ be the frequency at which $\nu B(\nu)$ is maximum.

Apparently, Bracewell's paper was not widely read by the astronomical community. As noted by Gehrels [2], the credit of introducing νf_ν to astronomers goes to Edward Ney, an eclectic physicist[1] who took up to IR astronomy in the 1960s. The first appearance of $\nu f_\nu = \lambda f_\lambda$ is a paper by Gehrz, Ney & Strecker [3] reporting mid-IR observations of hyper-giants.

It is now the standard lore that νf_ν captures "where most of the emission is occurs". The origin for this lore goes back to the blackbody intensity function. Consider the ratio

$$\alpha = \frac{\nu_{\rm p} B(\nu_{\rm p})}{\int B(\nu)d\nu} \tag{2}$$

where $\nu_{\rm p}$ is the frequency at which the Planck function, $B(\nu)$, peaks. We can define a similar ratio for $B(\lambda)$. Finally, for $\mathcal{F}$ we have $\alpha = \mathcal{F}(\nu'_{\rm p})/\int B(\nu)d\nu$. The corresponding photon energies are $E_{\rm p} = h\nu_{\rm p}$, $hc/\lambda_{\rm p}$ and $h\nu'_{\rm p}$. The photon energies linearly scale with k_BT. $E_{\rm p}/(k_BT)$ and α are given in the Table below.

[1] In 1940, as an undergrad at U. Minnesota, Ney, working with Professor Nier, showed that U-235 was responsible for fission rather than U-238, a key finding. After WWII, he switched to cosmic ray research. His experience in ballooning lead him to solar physics. His last academic foray was in aeronomy and astronomy: airglow, zodiacal light, intensity interferometry (with HB and T) and finally infrared astronomy. To further NIR astronomy he founded the O'Brien Observatory (close to Minneapolis) the Mt. Lemmon Observatory (in the Santa Catalina mountain range, Arizona). Nye seems to have produced a large number of outstanding students. Perhaps astronomers would recognize the names Danielson, Gehrz, Humphreys, Woolf, Rosen and Gillett amongst others. For more background about this interesting physicist see `https://en.wikipedia.org/wiki/Edward_P._Ney`.

From this table we see that the maxima of $\nu B(\nu)$ or $\lambda B(\lambda)$ indeed capture most of the integrated energy. However, the fraction of total energy captured by the maximum of $\mathcal{F}(\nu)$ is the largest. Gehrels [2] points out two incorrect statements about $\mathcal{F}$: it is not the energy within an octave and neither within a decade. It is the flux within a *logade*[2] (a factor of of $e \sim 2.7$) of frequency:

$$\mathcal{F}(\nu) = \nu \frac{dF}{d\nu} = \frac{dF}{d(\log(\nu))} \ .$$

Table 1: Blackbody

function	$E_{\rm p}/(k_B T)$	α
f_ν	2.851	62%
νf_ν	3.927	74%
f_λ	4.966	66%

In summary, the two principal benefits of using $\mathcal{F}(\nu)$ are best summarized by (1) Equation 1 (invariance of shape to the frequency spectral flux density or wavelength spectral flux density) and by (2) Equation 2, the ability of $\nu B(\nu)$ or $\lambda B)(\lambda)$ to capture the bolometric output (Table 1). Other advantages of using νf_ν: (1) the Sunyaev-Zeldovich effect disappears at the peak of νf_ν and (2) in cosmology, νf_ν transforms elegantly as D_L^{-2} with no additional factors.

3 Power Law Spectrum

Power laws are commonly used in astronomy.[3] We adopt the radio astronomy power-law model (in which the power law exponent does not carry a default minus sign):

$$f(\nu) = A_*(\nu/\nu_*)^\alpha \ . \tag{3}$$

Integrating this equation between ν_1 and ν_2 yields

$$I_\alpha(\nu_1, \nu_2) = A_* \nu_* \frac{1}{1+\alpha} \left[\left(\frac{\nu_2}{\nu_*} \right)^{\alpha+1} - \left(\frac{\nu_1}{\nu_*} \right)^{\alpha+1} \right].$$

[2]This word is my invention.

[3]The use of power law for broad-band emission, many times, is reasonable. Jokingly, one can say that power law is the first choice of scoundrels.

Evaluation of this Equation requires knowledge of A_*, ν_* and α. For the special case of $\alpha = -1$,

$$I_1(\nu_1, \nu_2) = \log(\nu_2/\nu_1) A_* \nu_* = \log(\nu_2/\nu_1) f(\nu) \nu \ .$$

The form of this Equation motivates us to define the following quantity as a surrogate for the integrated intensity:

$$G(\nu) \equiv \log(\nu_2/\nu_1) \nu f(\nu) = \log(\nu_2/\nu_1) A_* \nu_* (\nu/\nu_*)^{\alpha+1} \ . \tag{4}$$

Clearly the "bolometric" correction with respect to the surrogate is

$$C_\alpha(\nu) \equiv \frac{I_\alpha(\nu_1, \nu_2)}{G(\nu)} = \frac{1}{(1+\alpha)} \frac{1}{\log(\nu_2/\nu_1)} \left[\left(\frac{\nu_2}{\nu} \right)^{\alpha+1} - \left(\frac{\nu_1}{\nu} \right)^{\alpha+1} \right]. \tag{5}$$

The evaluation of C_α at the bounding frequencies, ν_1 and ν_2, and $\nu_g = (\nu_1 \nu_2)^{1/2}$, the geometric mean, yields:

$$\begin{aligned}
C_\alpha(\nu_1) &= \frac{1}{(1+\alpha)} \frac{1}{\log(\nu_2/\nu_1)} \left[\left(\frac{\nu_2}{\nu_1} \right)^{\alpha+1} - 1 \right], \\
C_\alpha(\nu_2) &= \frac{1}{(1+\alpha)} \frac{1}{\log(\nu_2/\nu_1)} \left[1 - \left(\frac{\nu_1}{\nu_2} \right)^{\alpha+1} \right], \\
C_\alpha(\nu_g) &= \frac{1}{(1+\alpha)} \frac{1}{\log(\nu_2/\nu_1)} \left[\left(\frac{\nu_2}{\nu_1} \right)^{\frac{\alpha+1}{2}} - \left(\frac{\nu_1}{\nu_1} \right)^{\frac{\alpha+1}{2}} \right].
\end{aligned} \tag{6}$$

The run of these correction factors with the α is shown in Figure 1.

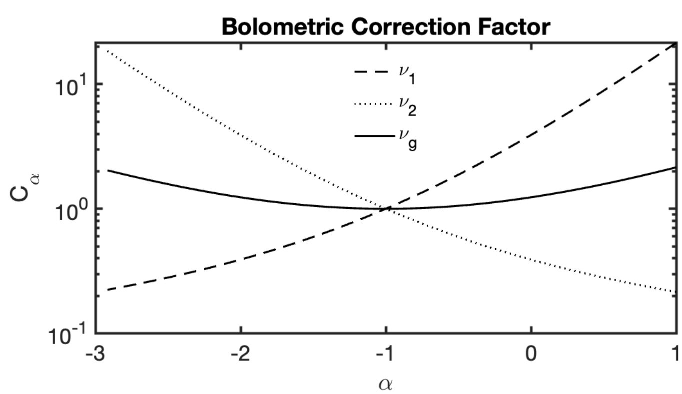

Figure 1: The bolometric factor, C_α, as a function of α. The frequency ratio, ν_2/ν_1 is set to 10.

Conclusion: The quantity

$$\nu_g F(\nu_g) \log(\nu_2/\nu_1) \tag{7}$$

where ν_g is the geometric mean of ν_1 and ν_2 is an excellent surrogate for the flux integrated between ν_1 and ν_2.

Equation 7 is useful when quoting fluxes from broad-band surveys, for example, X-ray imagers (2–10 keV or 0.2–2 keV) and also broad-band radio surveys (e.g., CHIME 400–800 MHz). In this case, it is best to quote the spectral flux density (Jy) by summing over the entire band and divide by $\nu_g \log(\nu_2/\nu_1)$ and attribute the resulting spectral flux density to ν_g. At least to my knowledge this modest result is an original conclusion.

4 X-ray and γ-ray Astronomy

X-ray astronomers use keV and sometimes the photon flux density which is the number of photons per unit time per unit area usually in a frequency bandwidth of $\nu_0 = h/E_0$ where E_0 =1 keV (X-ray astronomy) or 1 MeV (gamma-ray astronomy) and h is Planck's constant:

$$\frac{dN}{dE} = \frac{f(\nu)}{h\nu}\nu_0. \tag{8}$$

Beware of plotting line emission. Broad-band emission from the Galactic Center region is displayed in Figure 2. The e^+-e^- line seemingly has the same flux as the continuum emission at higher energies. However, the line is very narrow (primarily arising from annihilation in the interstellar medium. In this case, νf_ν is an over-estimate by the factor $\nu/\Delta\nu$ where $\Delta\nu$ is the width of the line.

REFERENCES

[1] R. Bracewell, Nature 174, pp 563 (1954)
[2] N. Gehrels, Ill Nuovo Cimento B 112, pp 11 (1997)
[3] R. D. Gehrz, E. P. Ney & Strecker, D. W., ApJ 161, pp L219 (1970)
[4] A. Toor & F. D. Seward, AJ 79, pp 995 (1974)

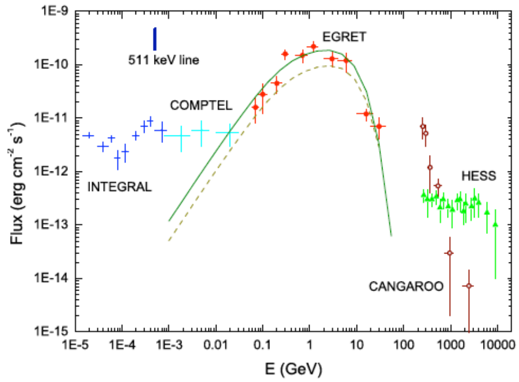

Figure 2: Broad band emission from the Galactic Center

A The Crab Nebula: milli-crab & micro-Jy

Traditionally, the Crab nebula is used as a calibrator for classical (2–10 keV) and hard X-ray band (10–100 keV). According to Toor & Seward [4]

$$I = AE^{-\Gamma}\exp(-\sigma N_H) \tag{9}$$

where $A = 9.7\,\mathrm{keV\,keV^{-1}\,cm^{-2}\,s^{-1}}$, $\Gamma = 1.1 \pm 0.03$ and the exponential term represents the ISM absorption. The sharp-eyed reader would have noticed that in X-ray astronomy the power law index comes with a default negative sign (the opposite of the model used by radio astronomers). The photon flux density is then $dN/dE \propto \nu^{-\Gamma_P}$ where $\Gamma_P = \Gamma + 1$, is one unit larger than Γ.

We find at $E = 1\,\mathrm{keV}$ (corresponding to $\nu = 2.4 \times 10^{17}\,\mathrm{Hz}$), $I = 6.4 \times 10^{-26}\,\mathrm{erg\,cm^{-2}\,s^{-1}\,Hz^{-1}}$. Thus, at $E = 6\,\mathrm{keV}$, the flux density is about 1 mJy whence the usual statement "1 Crab = 1 milliJy". Ignoring the exponential term, the flux between photon energy E_1 and E_2 (both in keV), given that α is almost 1, is

$$F = A\log(E_2/E_1)\,\mathrm{keV\,cm^{-2}\,s^{-1}}.$$

Since $1\,\mathrm{keV}{=}1.6 \times 10^{-9}\,\mathrm{erg}$ we find

$$\begin{aligned} F(2{-}10\,\mathrm{keV}) &= 2.5 \times 10^{-8}\,\mathrm{erg\,cm^{-2}\,s^{-1}} \\ F(20{-}50\,\mathrm{keV}) &= 1.4 \times 10^{-8}\,\mathrm{erg\,cm^{-2}\,s^{-1}}. \end{aligned}$$