Searching for periodicity

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A PDF of Power Spectrum

We assume that we have a time series of counts (not count rates) resulting from equally space bins of duration, Δt. I will start by assuming that there is no periodicity in the input time series. Let x_k be the counts in bin k with $k = 0, 1, ..., n-1$ with $\langle x_k \rangle = \lambda$.

The discrete Fourier transform is computed on a grid of frequency, $f = q\Delta f$ where $\Delta f = T^{-1}$ with $T = n\Delta t$; $q = 0, 1, ..., n/2$. The special frequency bins are $q = 0$ which is merely the sum of the time series and the Nyquist bin, which is the highest frequency (and also, like the dc bin, is real).The discrete Fourier transform, is given by the sum of the cosine and sine transforms:

$$F_q = S_q + jC_q \tag{1}$$

$$C_q = \sum_{k=0}^{n-1} x_k a_{k,q} \qquad S_q = \sum_{k=0}^{n-1} x_k b_{k,q}$$

where $\exp(j2\pi kq/N) = a_{k,q} + jb_{k,q}$. C_q and S_q, thanks to the Central Limit Theorem, are Gaussian variates. The mean values, $\langle C_q \rangle$ and $\langle S_q \rangle$, apart from the dc bin, are zero because by construction we assume that the input time series has no periodicity. The variance of, say C_q, is

$$\begin{aligned} V(C_q) &\equiv \langle C_q^2 \rangle = \sum_{k,k'} \langle x_k x_{k'} \rangle a_{k,q} a_{k',q} \\ &= \sum_{k,k'} (\lambda^2 + \delta_{k,k'}\lambda) a_{k,q} a_{k',q} \\ &= \frac{n}{2}(\lambda + \lambda^2) \end{aligned} \tag{2}$$

where we took advantage of the fact that the measurements are independent and thus uncorrelated and thus $\langle x_k x_{k'} \rangle = \lambda^2 + \lambda\delta_{k',k}$ and that $\sum a_{k,q} a_{k',q} = 0$ for $k \neq k'$, provided that $\Delta f = 1/T$ (which is the case). The power spectrum is,

$$P_S(q) = \langle F_q F_q^* \rangle = \langle C_q^2 + S_q^2 \rangle \tag{3}$$

given that S_q and C_q are zero-mean Gaussian variates with mean and, $P_S(q)$ follows exponential distribution (χ^2_2).

Let us define $z = P_S(q)/(n(\lambda + \lambda^2)/2)$. The probability distribution function of z is

$$p(z) = \exp(-z). \tag{4}$$

The mean and the variance of z is 1 and 1, respectively. Consider a power spectrum with n frequency bins. Let z_m be the maximum value in this series. The probability density function of z_m is then

$$p(z_m) = np(z_m)P(z_m)^{n-1} \tag{5}$$

where $P(z) = \int_0^z p(z)dz$ is the cumulative function of z. A plot of $p(z_m)$ and *its* cumulative function is shown in Figure 1. For $n = 8192$, in order to assure of a firm detection, say 99%, you would need $z \approx 13$.

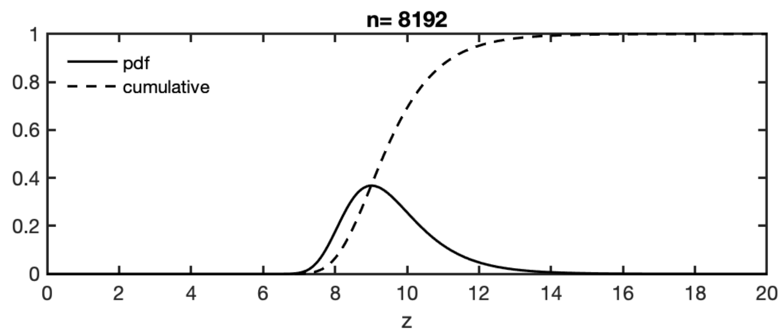

Figure 1: $p(z_m)$ (solid) and $\int p(z_m)dz_m$ (dashed) for $n = 8192$.