

The Physics of Polarization

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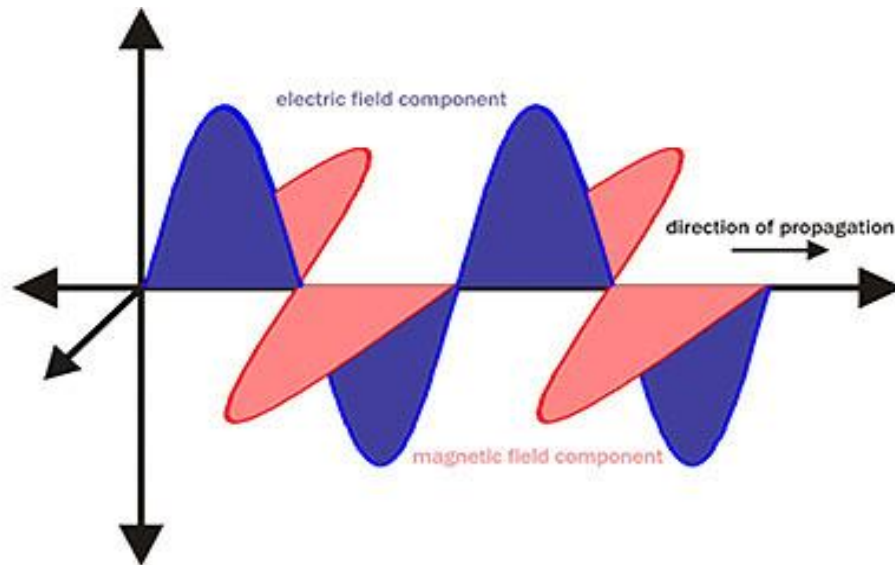
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Introduction I

Polarization is an important physical property of electromagnetic waves which is connected with the transversality character, with respect to the direction of propagation, of the electric and magnetic field vectors.



Introduction II

Rasmus Bartholin (Erasmus Bartholinus) was the first to report on a physical effect based on what we call today “polarization”. He wrote a treatise entitled “Experiments on double-refracting Icelandic crystals showing amazing and unusual refraction”, where he presents the astonishing properties of a calcite crystal.



Introduction III

Reflecting on Bartholin's experiments, Christiaan Huyghens in his "Threatyse on light" and Isaac Newton in his "Optics", though working within the framework of two completely different theories of light (undulatory and corpuscular, respectively) arrived to the conclusion that light should have some "transversality" property (not yet called polarization).

Newton



Huyghens



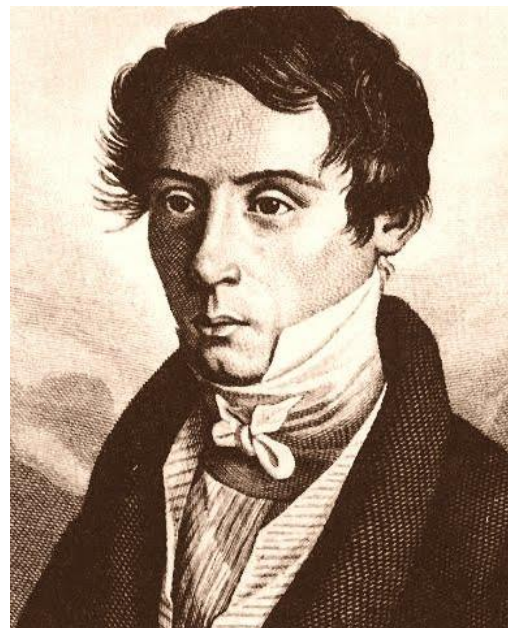
Introduction IV

After many years from Huyghens and Newton, the french physicist Etienne Louis Malus that introduces in the scientific literature the word “polarization”. In his paper “Sur une propriété de la lumière réfléchie” (1809) Malus proves that polarization is an intrinsic property of light and demonstrates that polarization can be easily produced through the phenomena of reflection and refraction. He also proves the famous $\cos^2\theta$ law today known as “Malus law”.



Introduction V

The work of Malus opens the way to the achievements of another french physicist, probably the most renowned optician of all times, Augustin Fresnel, who definitely proves the transversality of light. In his work “Mémoires sur la réflexion de la lumière polarisée”, Fresnel proves his famous laws concerning the polarization properties of light reflected and transmitted at the surface of a dielectric.



Introduction VI

The story of polarization goes on with several significant contributions by François Arago, Jean-Baptiste Biot (optical activity in crystals), David Brewster (today known for the “Brewster angle”), William Nicol (who builds the first polarizer, the so-called “Nicol prism”), and Michael Faraday (who discovers the “Farady effect”).



Arago



Brewster



Nicol (right)



Fraday

Introduction VII

However, it is only in 1852 with the fundamental work of George Stokes entitled “On the composition and resolution of streams of polarized light from different sources”, that the description in mathematical terms of polarized radiation becomes fully consistent. Stokes introduces four quantities, today known as the “Stokes parameters”, that have proven to be quite successful for the description of the polarization properties of a radiation beam.



Introduction VIII

At the middle of the XIX century, the phenomenon of polarization is thus fairly well understood, but it is necessary to wait for more than 50 years before the first astronomical application of polarization. In 1908 Hale succeeds in observing the spectrum of a sunspot in two opposite directions of circular polarization and, from the observed shift of the spectral lines deduces the existence of a magnetic field on an astronomical object, the sun.



Description of polarization I

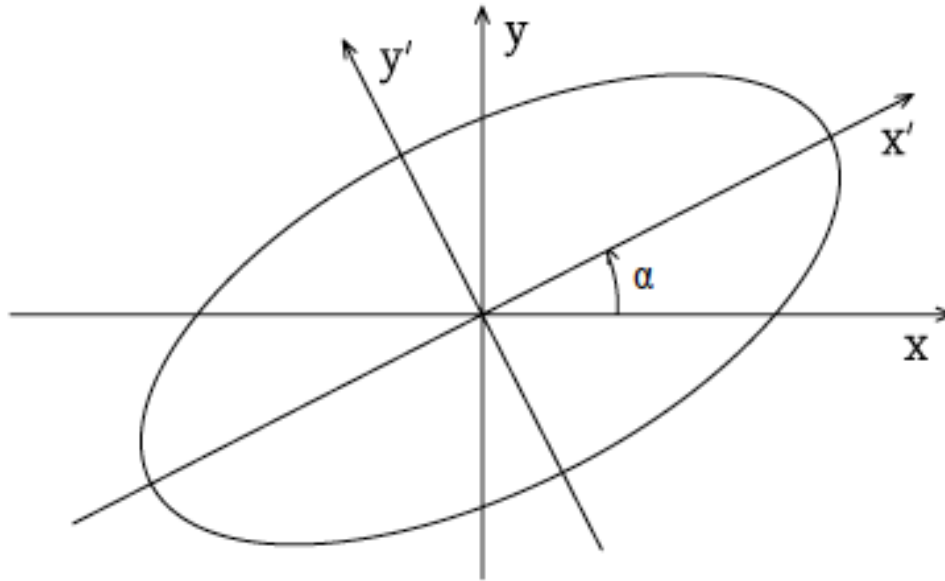
Consider an electromagnetic, monochromatic plane wave of angular frequency ω that is propagating in vacuum along a direction that we assume as the z-axis of a right-handed reference system. At a given point of space, the electric and magnetic field vectors of the wave oscillate in the x-y plane according to equations of the form

$$E_x(t) = E_1 \cos(\omega t - \phi_1)$$

$$E_y(t) = E_2 \cos(\omega t - \phi_2)$$

where E_1 , E_2 , ϕ_1 , and ϕ_2 are constants. These oscillations combine in such a way that the tip of the electric field vector describes an ellipse.

Description of polarization II



Obviously, the polarization ellipse can degenerate into a segment (and one then speaks of **linear polarization**) or it can degenerate into a circle (and one then speaks of **circular polarization**).

Description of polarization III

The description now given is however valid only for a plane, monochromatic wave which goes on indefinitely from $t = -\infty$ to $t = \infty$. This is obviously a mathematical abstraction which, in general, has little to do with the physical world. A much more realistic description of a beam of radiation can be given only in terms of a statistic superpositions of many wave-packets each having a limited extension in space and time. The beam thus loses its property of being monochromatic, becoming a quasi-monochromatic wave. Moreover, if the individual wave-packets do not share the same polarization properties, the polarization ellipse varies, statistically, in time.

Description of polarization IV

The statistical description of polarization, introduced by Stokes, implies considering the statistical averages of bilinear products of the electric field components along the x and y axes and forming from them four independent linear combinations:

$$P_I = \langle E_1^2 + E_2^2 \rangle = \langle \mathcal{E}_1^* \mathcal{E}_1 \rangle + \langle \mathcal{E}_2^* \mathcal{E}_2 \rangle \quad ,$$

$$P_Q = \langle E_1^2 - E_2^2 \rangle = \langle \mathcal{E}_1^* \mathcal{E}_1 \rangle - \langle \mathcal{E}_2^* \mathcal{E}_2 \rangle \quad ,$$

$$P_U = \langle 2E_1 E_2 \cos(\phi_1 - \phi_2) \rangle = \langle \mathcal{E}_1^* \mathcal{E}_2 \rangle + \langle \mathcal{E}_2^* \mathcal{E}_1 \rangle \quad ;$$

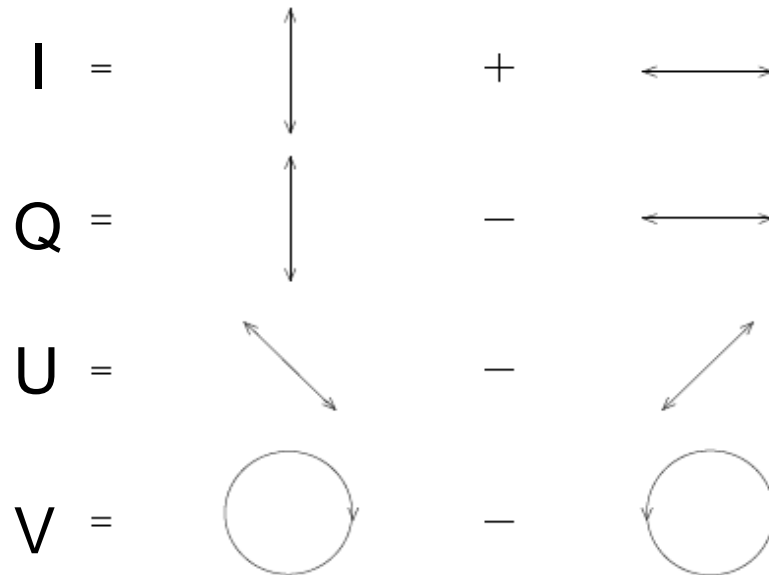
$$P_V = \langle 2E_1 E_2 \sin(\phi_1 - \phi_2) \rangle = i (\langle \mathcal{E}_1^* \mathcal{E}_2 \rangle - \langle \mathcal{E}_2^* \mathcal{E}_1 \rangle)$$

where

$$e \quad \mathcal{E}_1 = E_1 e^{i\phi_1} \quad , \quad \mathcal{E}_2 = E_2 e^{i\phi_2} \quad .$$

Description of polarization V

The quantities now introduced are called the Stokes parameters. This is a pictorial representation and, at the same time, an operational definition



Through the Stokes parameters it is possible to give a coherent definition of the polarization properties of an arbitrary beam of radiation.

Polarization and optical devices I

The operational definition given above implies the use of two fundamental optical devices: the polarizer and the retarder. From the polarimetric point of view any optical device is characterized by a 2 x 2 matrix of the form

$$\begin{pmatrix} \mathcal{E}'_1 \\ \mathcal{E}'_2 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \mathcal{E}_1 \\ \mathcal{E}_2 \end{pmatrix}$$

relating the complex electric field components of the exit beam to the corresponding components of the input beam. The matrix appearing in this equation is called a Jones matrix. For a train of N optical components, the Jones matrix of the train is given by the product of N Jones matrices.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a_N & b_N \\ c_N & d_N \end{pmatrix} \cdots \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$$

Polarization and optical devices II

In particular, for an ideal polarizer one has

$$\begin{pmatrix} \mathcal{E}'_a \\ \mathcal{E}'_b \end{pmatrix} = e^{i\psi} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \mathcal{E}_a \\ \mathcal{E}_b \end{pmatrix} = e^{i\psi} \begin{pmatrix} \mathcal{E}_a \\ 0 \end{pmatrix}$$

so that a single component of the electric field survives. Whereas for an ideal retarder

$$\begin{pmatrix} \mathcal{E}'_f \\ \mathcal{E}'_s \end{pmatrix} = e^{i\psi} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} \mathcal{E}_f \\ \mathcal{E}_s \end{pmatrix} = e^{i\psi} \begin{pmatrix} \mathcal{E}_f \\ e^{i\delta} \mathcal{E}_s \end{pmatrix}$$

The two components of the electric field along the directions of the “fast” and the “slow” axis are dephased. The phase difference δ is called the retardance. When $\delta = \pi/4$, the retarder is called a *quarter-wave plate*.

Polarization and optical devices III

The relationship between the electric field components of the entrance and exit beams expressed by a Jones matrix given can be translated into a relationship between the Stokes parameters. From their definition one gets

$$\mathbf{S}' = \mathbf{M} \mathbf{S}$$

where \mathbf{S} is a 4-component vector constructed with the Stokes parameters of the entrance beam, \mathbf{S}' has a similar meaning for the exit beam, and \mathbf{M} is a 4x4 matrix given by

$$\frac{1}{2} \begin{pmatrix} a^*a + b^*b + c^*c + d^*d & a^*a - b^*b + c^*c - d^*d & 2\text{Re}(a^*b + c^*d) & 2\text{Im}(a^*b + c^*d) \\ a^*a + b^*b - c^*c - d^*d & a^*a - b^*b - c^*c + d^*d & 2\text{Re}(a^*b - c^*d) & 2\text{Im}(a^*b - c^*d) \\ 2\text{Re}(a^*c + b^*d) & 2\text{Re}(a^*c - b^*d) & 2\text{Re}(a^*d + b^*c) & 2\text{Im}(a^*d - b^*c) \\ -2\text{Im}(a^*c + b^*d) & -2\text{Im}(a^*c - b^*d) & -2\text{Im}(a^*d + b^*c) & 2\text{Re}(a^*d - b^*c) \end{pmatrix}$$

Polarization and optical devices IV

The 4 x 4 matrix M introduced in the previous slide is a particular case of a so-called Muller matrix. For a train of N optical devices, each characterized by its own Muller matrix, one can construct the Muller matrix of the train by simply multiplying the different matrices

$$\mathbf{M} = \mathbf{M}_N \dots \mathbf{M}_2 \mathbf{M}_1$$

The advantage of Muller matrices with respect to the Jones matrices is that the first ones are capable of describing depolarization phenomena, while the second are not. For instance, the Muller matrix of an ideal depolarizer is

$$\mathbf{M}_{\text{ideal depolarizer}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Polarization and optical devices V

The calculus based on Jones matrices and Mueller matrices has a large variety of applications in physics and, more particularly, in astronomy. It is at the base of the design of polarimeters operating in the different regions of the electromagnetic spectrum, from the ultraviolet to the infrared. In many cases, one can even define the Mueller matrix of a telescope by analyzing the properties of each of its optical devices and then deducing the resulting matrix as the product of the matrices of each device. The knowledge of the Muller matrix of a telescope is essential for a proper calibration of the polarization observed in the focal plane of a telescope.

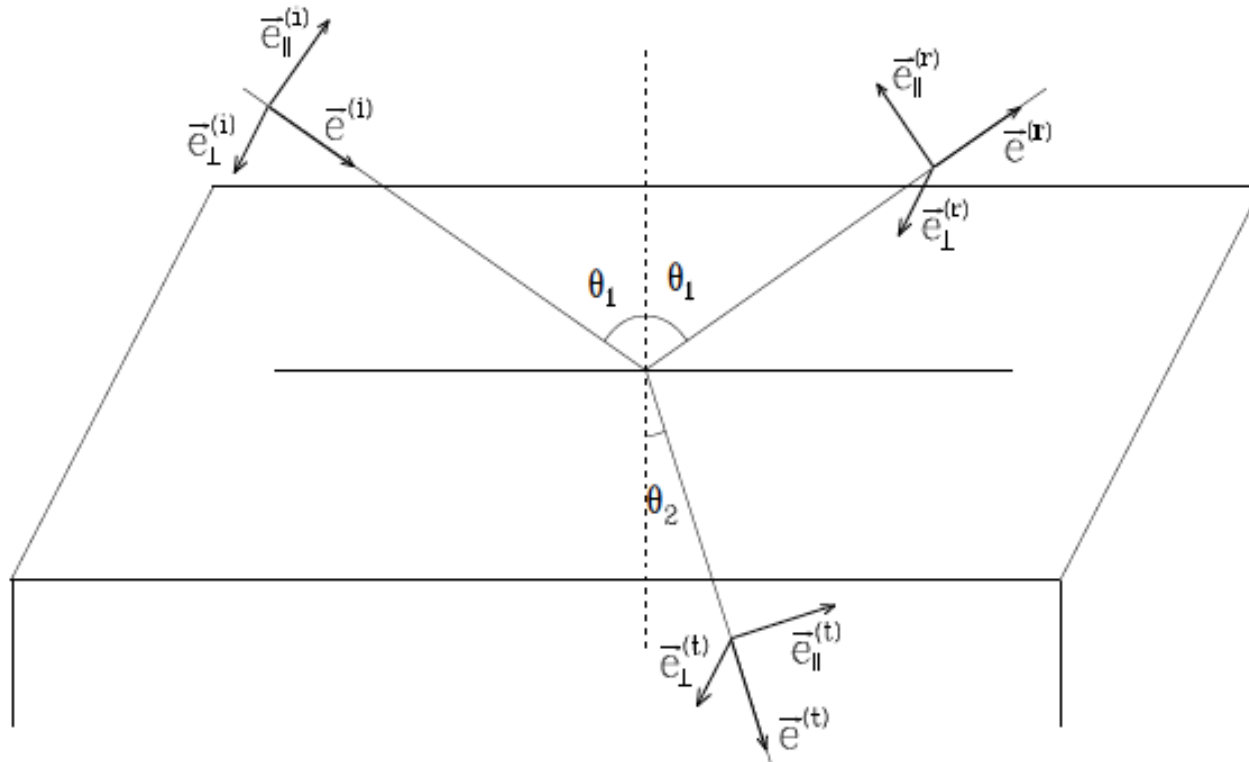
Polarization and physical phenomena

Reflection and refraction I

The simplest and most common physical phenomenon where polarization processes enter into play is the ordinary reflection of a pencil of radiation on the surface of a dielectric (or of a metallic) medium. This phenomenon, which is generally accompanied by the related phenomenon of refraction inside the medium, is described by the so-called Fresnel equations. In a modern approach these equations can be derived from Maxwell's equations, though Fresnel, obviously, had no idea of them
(Maxwell was Born 4 years after the death of Fresnel).

Polarization and physical phenomena

Reflection and refraction II



In the reflection and refraction of the beam of radiation, the polarization properties are deeply modified. In terms of Jones matrices one has

Polarization and physical phenomena

Reflection and refraction III

Reflected beam

$$\begin{pmatrix} \mathcal{E}_{\parallel}^{(r)} \\ \mathcal{E}_{\perp}^{(r)} \end{pmatrix} = \begin{pmatrix} r_{\parallel} & 0 \\ 0 & r_{\perp} \end{pmatrix} \begin{pmatrix} \mathcal{E}_{\parallel}^{(i)} \\ \mathcal{E}_{\perp}^{(i)} \end{pmatrix}$$

Refracted beam

$$\begin{pmatrix} \mathcal{E}_{\parallel}^{(t)} \\ \mathcal{E}_{\perp}^{(t)} \end{pmatrix} = \begin{pmatrix} t_{\parallel} & 0 \\ 0 & t_{\perp} \end{pmatrix} \begin{pmatrix} \mathcal{E}_{\parallel}^{(i)} \\ \mathcal{E}_{\perp}^{(i)} \end{pmatrix}$$

$$r_{\parallel} = \frac{n_2 \cos \theta_1 - n_1 \cos \theta_2}{n_2 \cos \theta_1 + n_1 \cos \theta_2}, \quad r_{\perp} = \frac{n_1 \cos \theta_1 - n_2 \cos \theta_2}{n_1 \cos \theta_1 + n_2 \cos \theta_2}$$

$$t_{\parallel} = \frac{2n_1 \cos \theta_1}{n_2 \cos \theta_1 + n_1 \cos \theta_2}, \quad t_{\perp} = \frac{2n_1 \cos \theta_1}{n_1 \cos \theta_1 + n_2 \cos \theta_2}$$

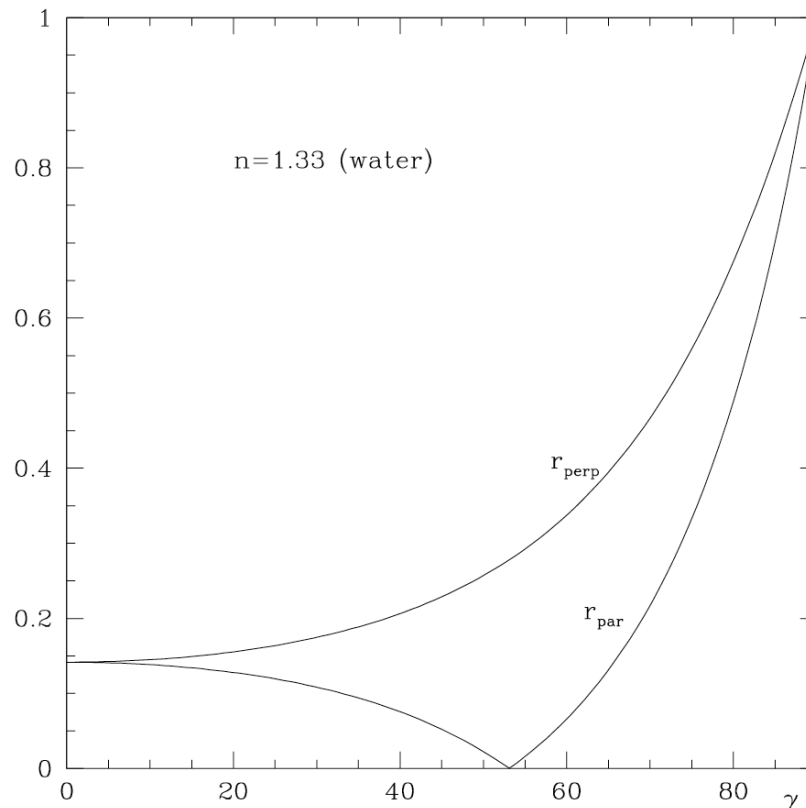
with

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad (\text{Snell's law})$$

Polarization and physical phenomena

Reflection and refraction IV

For reflection on water, for instance, the coefficients are shown in this graph. Note that for a particular angle (the Brewster angle) $r_{\text{par}} = 0$. The reflected beam is totally polarized



Polarization and physical phenomena

Reflection and refraction V



(a)

(b)

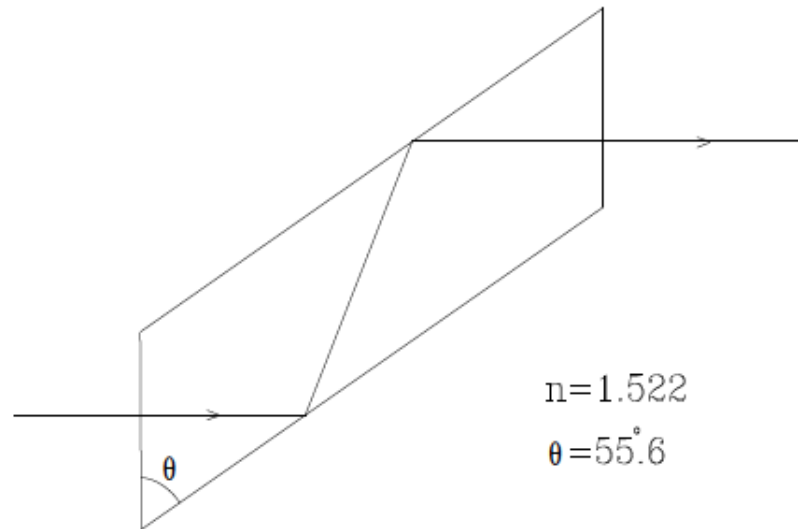
Left image: without polarizing filter

Right image: with polarizing filter

Polarization and physical phenomena

Reflection and refraction VI

Radiation propagating from inside a dielectric to air can suffer the phenomenon of total internal reflection. In such reflection a dephasing occurs between the parallel and perpendicular components of the incident electric field. This allows to construct an instrument capable of transforming linear into circular polarization, such as the Fresnel rhomb.



Polarization and physical phenomena

Dichroism and anomalous dispersion I

Though the etymology of the word is rather misleading, dichroism is a typical phenomenon of anisotropic media which consists in the fact that the absorption properties of the radiation propagating inside such media depends on polarization. Anomalous dispersion is a related phenomenon due to the dephasing of the two components of the electric field vector.

The two phenomena can be “unified” by thinking about the two complex amplitudes E_a and E_b of the electric field along two orthogonal states of polarization. Dichroism is connected with the differential attenuation of the modulus of E_a and E_b during the propagation, whereas anomalous dispersion is connected with the dephasing of the same quantities.

Polarization and physical phenomena

Dichroism and anomalous dispersion II

A description of these phenomena for a general anisotropic medium can be given by introducing the principal axes of the medium, characterized by three unit vectors \mathbf{u}_α ($\alpha=1, 2, 3$), and the corresponding indexes of refraction n_α , which are, in general, complex numbers. A wave of angular frequency ω , polarized along the direction \mathbf{u}_α , propagates within the medium according to the equation

$$\vec{E}(\vec{x}, t) = E_\alpha \vec{u}_\alpha e^{i\frac{\omega}{c}(n_\alpha \vec{x} \cdot \vec{\Omega} - ct)}$$

By considering the bilinear products of the field components, with some algebra it is possible to write a transfer equation for the Stokes parameters encompassing the phenomena of dichroism and anomalous dispersion.

Polarization and physical phenomena

Dichroism and anomalous dispersion III

$$\frac{d}{ds} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = - \begin{pmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_V & \eta_I & \rho_Q \\ \eta_V & \rho_U & \rho_Q & \eta_I \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}$$

The symmetric part of the matrix describes dichroism whereas the antisymmetric part describes anomalous dispersion.

$$\eta_I = \operatorname{Re}(G_{11} + G_{22}), \quad \eta_Q = \operatorname{Re}(G_{11} - G_{22}),$$

$$\eta_U = \operatorname{Re}(G_{12} + G_{21}), \quad \eta_V = \operatorname{Im}(G_{12} - G_{21}),$$

$$\rho_Q = -\operatorname{Im}(G_{11} - G_{22}), \quad \rho_U = -\operatorname{Im}(G_{12} + G_{21})$$

$$\rho_V = \operatorname{Re}(G_{12} - G_{21})$$

Polarization and physical phenomena

Dichroism and anomalous dispersion IV

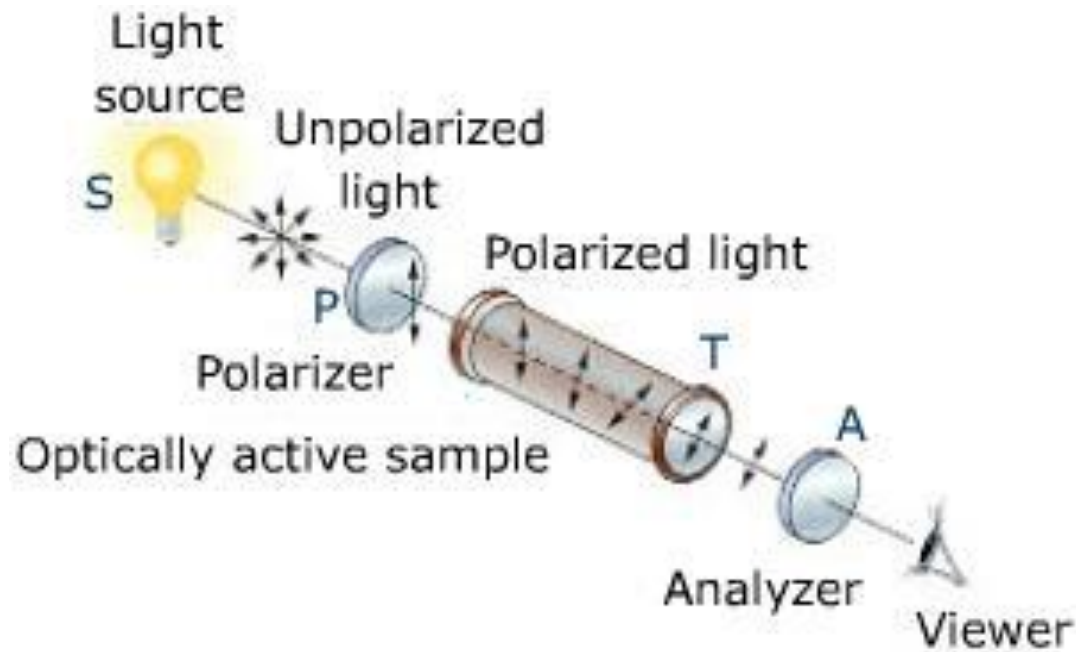
Where the quantities G are connected to the indices of refraction and to the unit vectors by the equation

$$G_{ij} = -i \frac{\omega}{c} \sum_{\alpha} n_{\alpha} (\vec{u}_{\alpha} \cdot \vec{e}_i) (\vec{u}_{\alpha}^* \cdot \vec{e}_j)$$

Particularly important is the term ρ_v appearing in the transfer equation of the previous slide. This term produces, in the propagation, the continuous transformation of the Stokes parameter Q in the Stokes parameter U and viceversa. The result is the rotation of the plane of polarization of light of a quantity which is proportional to the path-length.

Polarization and physical phenomena

Dichroism and anomalous dispersion V



Optical activity was discovered by Arago in quartz crystals, by Biot in solutions of inorganic and organic substances (including sugar) and by Faraday in media pervaded by a magnetic field (Faraday effect).

Polarization in everyday life I

Polarization is a rather obscure concept for the man on the street. This is not because polarization phenomena are not present in the world around us, but just because the human eye (differently from the eye of other living beings) is practically insensitive to the polarization of light, though some minor effects, due to the presence of a blue, dichroically absorbing pigment in the macula lutea can indeed be observed under particular conditions (Heidinger's brush).



Polarization in everyday life II

We have already seen that the light reflected from a dielectric surface is linearly polarized, the direction of polarization being the perpendicular to the plane of incidence. It then follows that sunlight reflected over the surface of the sea (or of a lake) is also linearly polarized and that such reflections can be extinguished, to a large extent, by means of a polarizing filter whose transmission axis is set along the vertical. This can be accomplished by wearing a particular type of sunglasses that can be commonly found in commerce and are usually referred to as Polaroids (Polaroid is a registered mark). The "lenses" of these sunglasses are nothing but polarizing filters suitably oriented by the manufacturer in such a way that the transmission axis coincides with the vertical axis.

Polarization in everyday life III

With such sunglasses one can easily observe that the blue sky is strongly polarized, a phenomenon that is known since a long time and that is due to the scattering of sunlight by the air molecules. As we will see later, scattering by air molecules obeys the Rayleigh scattering law, which implies that, for a 90° scattering, the polarization of the scattered radiation is 100% linearly polarized, and that the direction of polarization is perpendicular to the scattering plane.



Polarization in everyday life IV

The polarization of the blue sky is an important physical phenomenon that is indeed used by several living beings (some species of insects, in particular) as a practical mean of orientation. Obviously, these insects are provided of a particular kind of eyes which allow them to observe the direction of polarization of the sky and to recover the sun's position. It has also been suggested that the navigators of Viking ships used a piece of Iceland spar to help them in finding the sun's direction in the heavily cloudy, northern skies of Scandinavia.

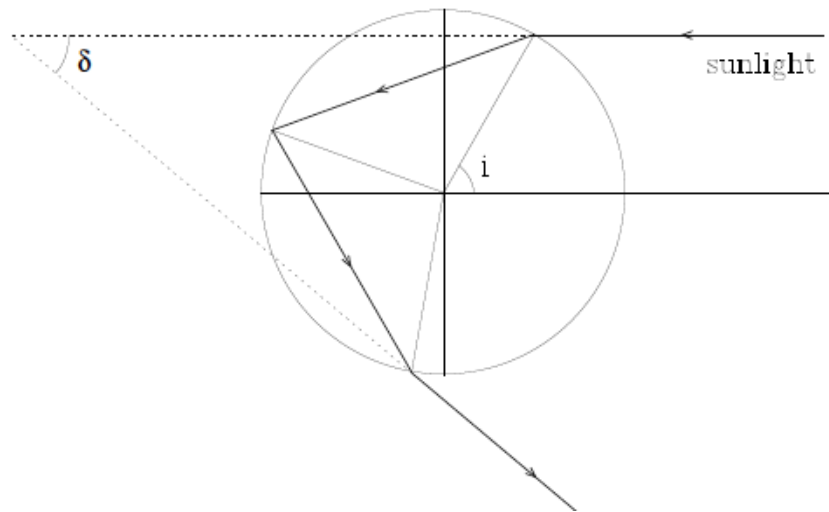


Polarization in everyday life V

Polarization also shows up in other meteorological phenomena, like in the rainbow and in halos. The rainbow is produced by the refraction of the solar radiation by droplets of water. The primary arch is due to a process of refraction (the solar rays enter the droplet), an internal reflection), and a final refraction (by which the solar ray exits the droplet). An analysis based on the Fresnel equations allows to deduce that the rainbow is linearly polarized, the polarization direction being parallel to the bow, and the fractional polarization being given by

$$P_{\text{rainbow}} = \frac{(2 + n^2)^6 - 729 n^4 (2 - n^2)^2}{(2 + n^2)^6 + 729 n^4 (2 - n^2)^2}$$

Polarization in everyday life VI



$$\delta \approx 42^\circ$$

Substituting the value of n , it is found that the polarization is parallel to the arch and is of the order of 93%.



Polarization in everyday life VII

The halo, a somewhat less known phenomenon which shows up when ice crystals are present in the upper atmosphere, is due to the refraction of sunlight inside the crystals themselves. It shows up as a luminous circle around the sun located at a distance of about 22° . The ice crystals have the shape of long prisms having an hexagonal cross section. A simple application of the Fresnel equations shows that the halo is linearly polarized, the polarization being directed perpendicularly to the halo, and the fractional polarization being given by

$$P_{\text{halo}} = \frac{(n\sqrt{3} + \sqrt{4 - n^2})^4 - (\sqrt{3} + n\sqrt{4 - n^2})^4}{(n\sqrt{3} + \sqrt{4 - n^2})^4 + (\sqrt{3} + n\sqrt{4 - n^2})^4}$$

Polarization in everyday life VIII

Substituting the value of the index of refraction of ice ($n=1.31$) one finds that the halo is much less polarized than the rainbow (only 4%). The direction of polarization is perpendicular to the halo.



Polarization in everyday life IX

Finally, it is important to remark that polarimetry has a very large number of technological applications in many practical aspects of life. An example is the use of polarimetric techniques to measure the quality of sugar. The sugar is diluted in a solution and the rotatory power of the solution is then measured through an instrument that is called a saccharimeter. From the rotatory power measured it is easy to recover the quality (saccharosium content) of the sugar contained in the solution. Another example concerns security systems that turn on when a beam of light is interrupted. By encoding the light beam and the receiver according to a particular state of polarization, it becomes almost impossible to substitute the ordinary beam with a second one having the same polarimetric characteristics.

Polarization due to radiating charges I

According to classical electrodynamics, an accelerated electric charge emits radiation into space. Starting from the Maxwell equations and introducing the so-called Liénard and Wiechert potentials, it is possible to show that for a non-relativistic charge e the electric and magnetic field of the emitted radiation is given, in the radiation zone at distance R and in the direction \mathbf{n} , by the equation

$$\vec{E}(\vec{r}, t) = \frac{e}{c^2 R} \vec{n} \times (\vec{n} \times \vec{a}(t'))$$

$$\vec{B}(\vec{r}, t) = \vec{n} \times \vec{E}(\vec{r}, t) \quad .$$

Where $\vec{a}(t')$ is the acceleration at the retarded time.

Polarization due to radiating charges II

The “recipe” to find the spectral and polarization properties of the radiation emitted along any direction by the moving charge proceeds by first decomposing the electric field along two orthogonal unit vectors \mathbf{e}_1 and \mathbf{e}_2 both perpendicular to the direction of propagation and then finding the Fourier components of such components E_1 and E_2

$$\mathcal{E}_1(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E_1(\vec{r}, t) e^{i\omega t} dt$$

$$\mathcal{E}_2(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E_2(\vec{r}, t) e^{i\omega t} dt$$

From the Fourier components one gets the Stokes parameters through the equation

Polarization due to radiating charges III

$$I_\omega = \mathcal{K} \left(\langle \mathcal{E}_1^*(\omega) \mathcal{E}_1(\omega) \rangle + \langle \mathcal{E}_2^*(\omega) \mathcal{E}_2(\omega) \rangle \right)$$

$$Q_\omega = \mathcal{K} \left(\langle \mathcal{E}_1^*(\omega) \mathcal{E}_1(\omega) \rangle - \langle \mathcal{E}_2^*(\omega) \mathcal{E}_2(\omega) \rangle \right)$$

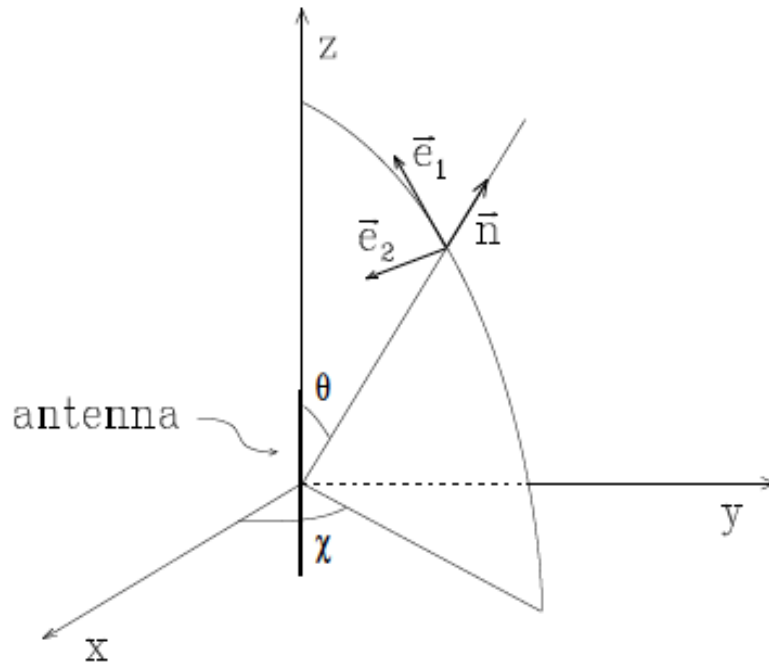
$$U_\omega = \mathcal{K} \left(\langle \mathcal{E}_1^*(\omega) \mathcal{E}_2(\omega) \rangle + \langle \mathcal{E}_2^*(\omega) \mathcal{E}_1(\omega) \rangle \right)$$

$$V_\omega = i \mathcal{K} \left(\langle \mathcal{E}_1^*(\omega) \mathcal{E}_2(\omega) \rangle - \langle \mathcal{E}_2^*(\omega) \mathcal{E}_1(\omega) \rangle \right)$$

In the following slides we give some examples relative to simple geometries for the motions of the radiating charges.

Polarization due to radiating charges IV

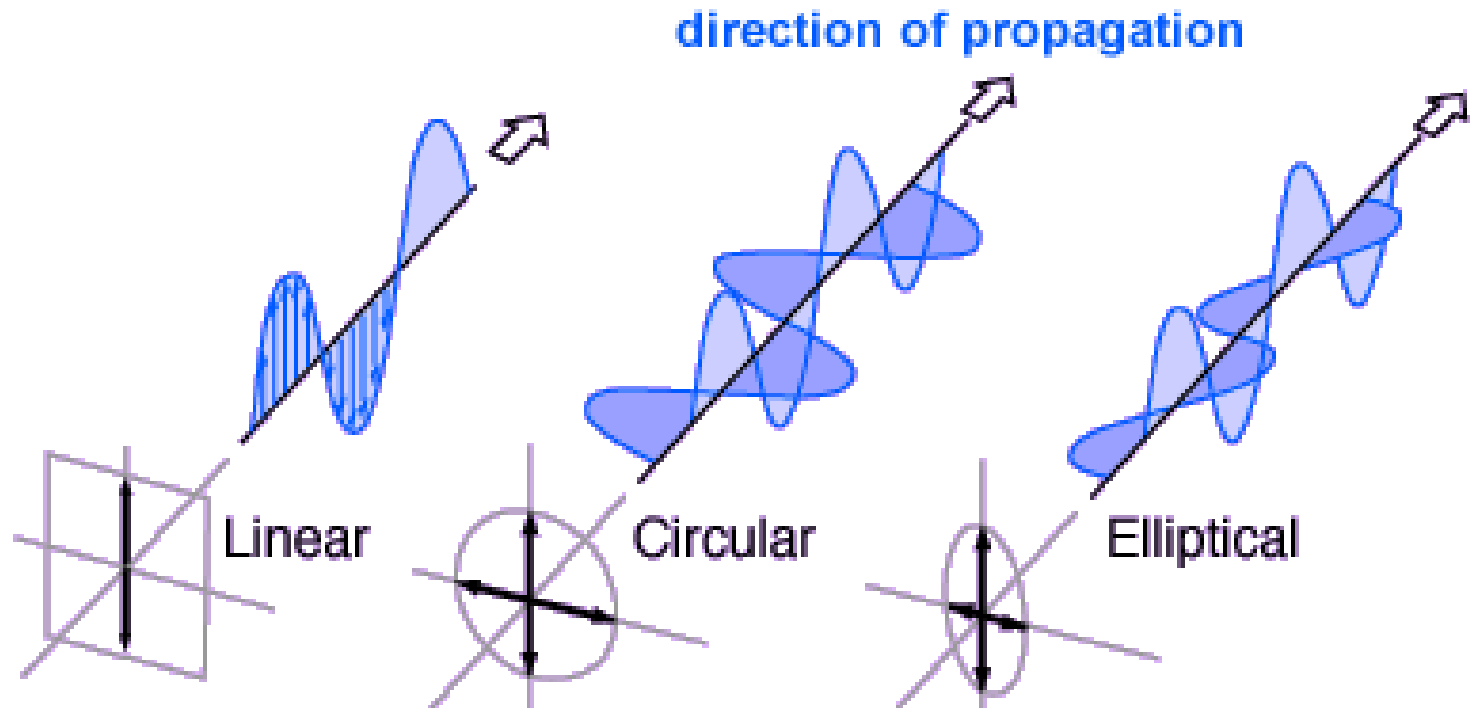
Linear antenna: charges oscillating along a fixed direction



The radiation is emitted at the same frequency of the oscillating charge (monochromatic radiation). The polarization is along the “meridian plane” (along the unit vector \mathbf{e}_1 of the figure).

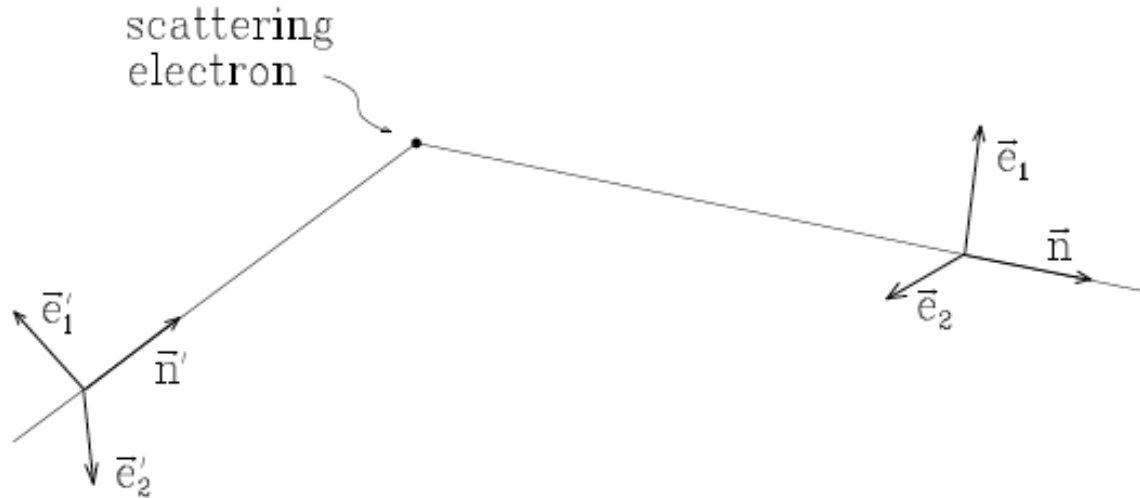
Polarization due to radiating charges V

Circular and elliptical antenna: the polarization of the radiation emitted by the antenna matches the motion of the oscillating charge.



Polarization due to radiating charges VI

Thomson scattering of radiation on free electrons



The components of the electric field of the scattered radiation, E_1 and E_2 , are connected to those of the incident radiation, E'_1 and E'_2 , by the equation

$$\begin{pmatrix} \mathcal{E}_1 \\ \mathcal{E}_2 \end{pmatrix} = -\frac{r_c}{R} \begin{pmatrix} \vec{e}_1 \cdot \vec{e}'_1 & \vec{e}_1 \cdot \vec{e}'_2 \\ \vec{e}_2 \cdot \vec{e}'_1 & \vec{e}_2 \cdot \vec{e}'_2 \end{pmatrix} \begin{pmatrix} \mathcal{E}'_1 \\ \mathcal{E}'_2 \end{pmatrix}$$

Polarization due to radiating charges VII

Translated into the Stokes parameters formalism the previous equation results

$$S(\vec{n}) = \frac{2r_c^2}{3R^2} \mathbf{R}(\vec{n}, \vec{n}') S'(\vec{n}')$$

Where the 4x4 matrix R (the Rayleigh matrix) is

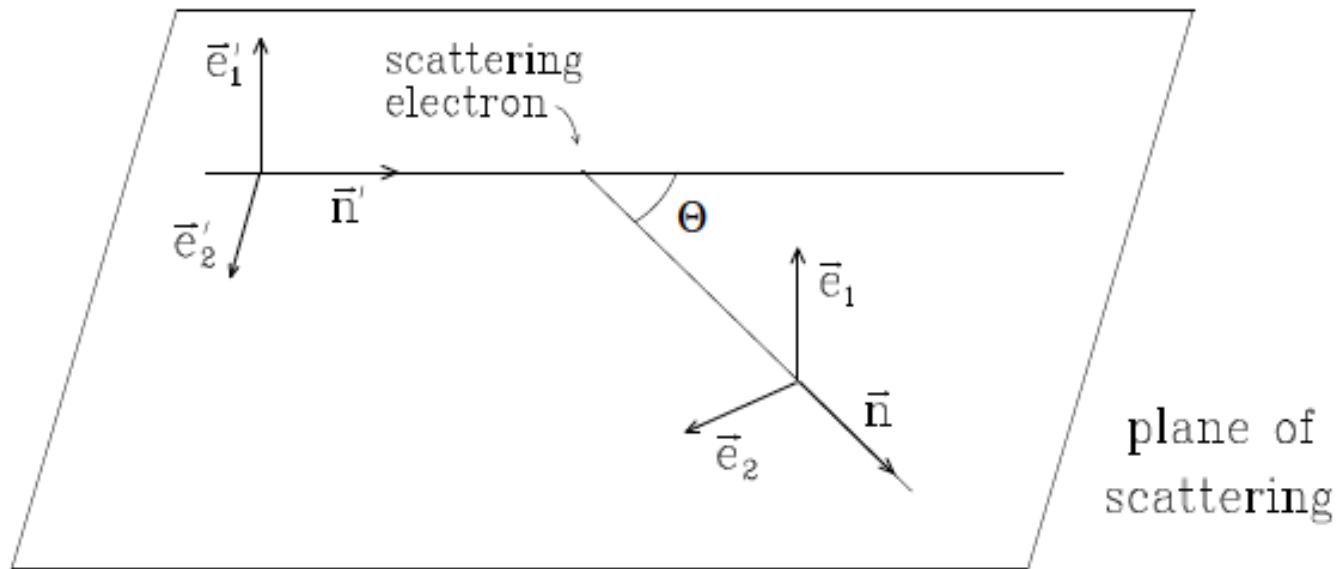
$$\mathbf{R}(\vec{n}, \vec{n}') = \frac{3}{4} \begin{pmatrix} a^2 + b^2 + c^2 + d^2 & a^2 - b^2 + c^2 - d^2 & 2(ab + cd) & 0 \\ a^2 + b^2 - c^2 - d^2 & a^2 - b^2 - c^2 + d^2 & 2(ab - cd) & 0 \\ 2(ac + bd) & 2(ac - bd) & 2(ad + bc) & 0 \\ 0 & 0 & 0 & 2(ad - bc) \end{pmatrix}$$

with

$$a = \vec{e}_1 \cdot \vec{e}'_1, \quad b = \vec{e}_1 \cdot \vec{e}'_2, \quad c = \vec{e}_2 \cdot \vec{e}'_1, \quad d = \vec{e}_2 \cdot \vec{e}'_2.$$

Polarization due to radiating charges VIII

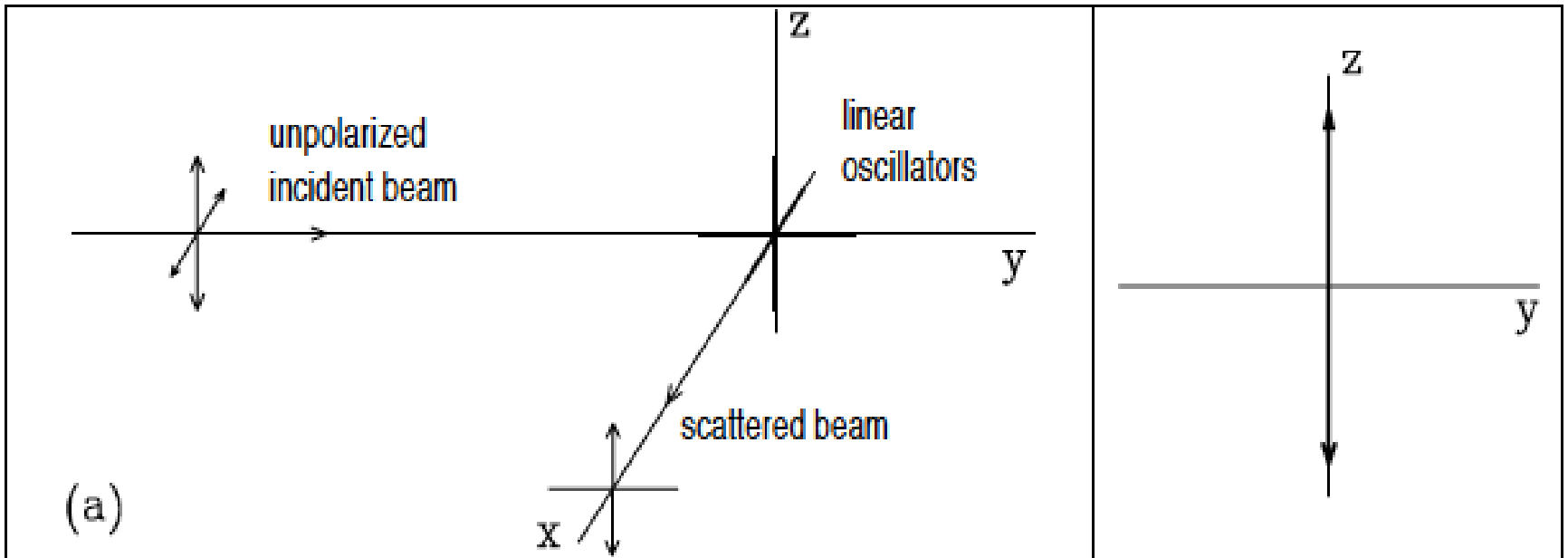
With a suitable choice of the polarization unit vectors (in the “good geometry”) the matrix simplifies



$$\mathbf{R}(\Theta) = \frac{3}{4} \begin{pmatrix} 1 + \cos^2 \Theta & \sin^2 \Theta & 0 & 0 \\ \sin^2 \Theta & 1 + \cos^2 \Theta & 0 & 0 \\ 0 & 0 & 2 \cos \Theta & 0 \\ 0 & 0 & 0 & 2 \cos \Theta \end{pmatrix}$$

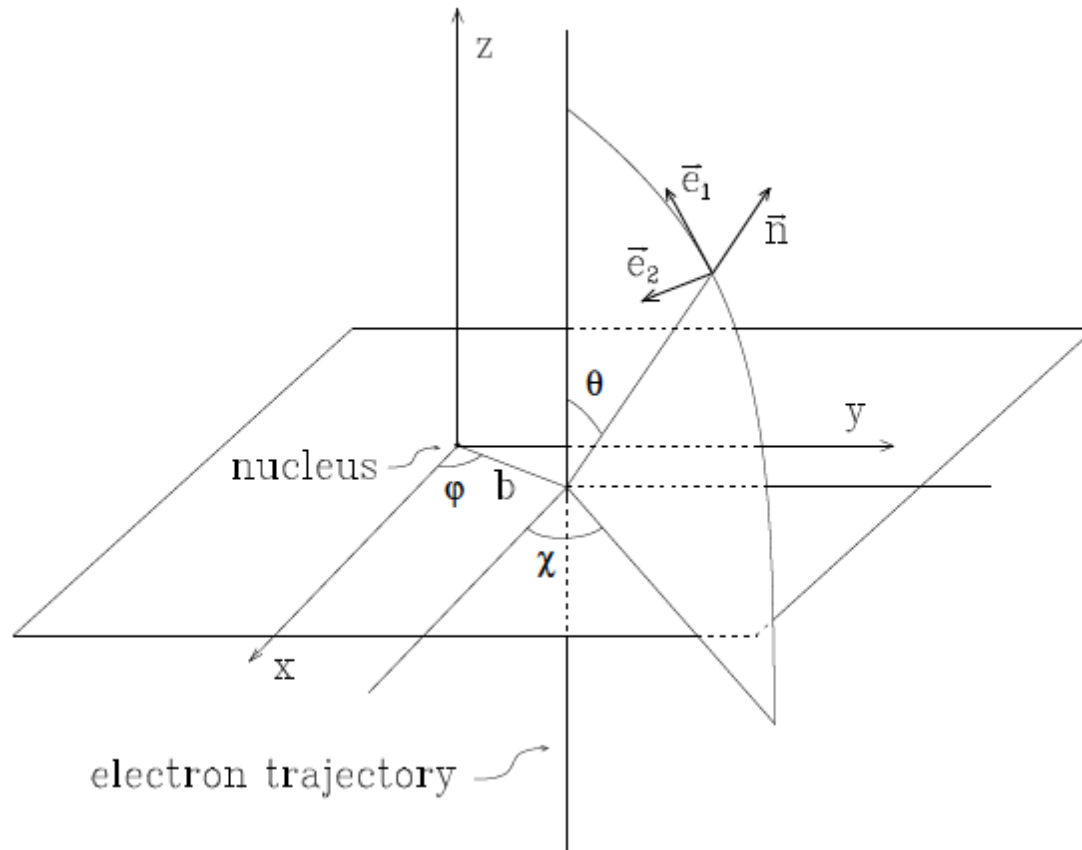
Polarization due to radiating charges IX

In particular, for a scattering angle of 90° , the scattered radiation is linearly polarized independently of the polarization of the incident radiation. This behavior can be easily understood in terms of linear oscillators.



Polarization due to radiating charges X

The radiation emitted in the process of collision between a fast electron and a heavy nucleus is usually referred to under the name of **bremsstrahlung radiation**.



Polarization due to radiating charges XI

For a unidirectional beam of electrons, all having the same direction, the emitted radiation is polarized. Approximately, the polarization is given by

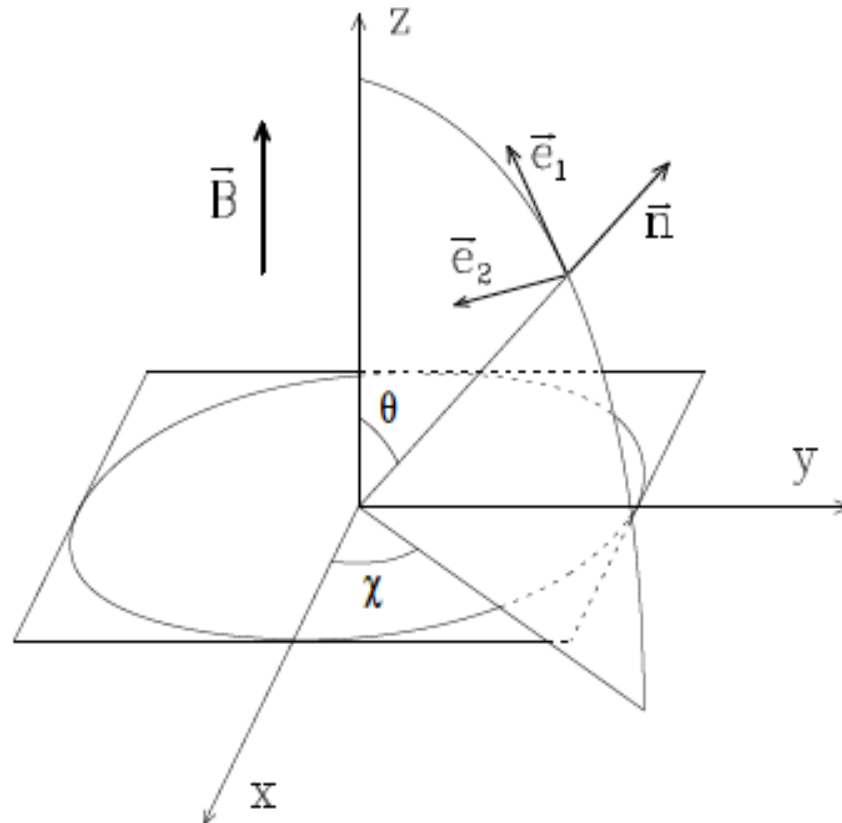
$$\frac{Q_{\omega}(\vec{n})}{I_{\omega}(\vec{n})} = - \frac{\sin^2 \theta}{1 + \cos^2 \theta}$$

The minus sign in this equation implies that the polarization is perpendicular to the direction of the incident electrons. The equation shows that the maximum polarization is obtained for $\theta = \pi/2$ (radiation emitted perpendicularly to the direction of the incident beam). For $\theta = 0$ or π the polarization is zero. In astrophysical plasmas, however, it is difficult to find anisotropic beam of electrons. For isotropic collisions the polarization obviously disappears.

Polarization due to radiating charges XII

A non-relativistic electron moving under the action of a constant and uniform magnetic field describes a helical motion whose projection on the plane perpendicular to the magnetic field direction is a circle. This circular motion has a typical frequency called the cyclotron frequency

$$\omega_C = \frac{e_0 B}{mc}$$



Polarization due to radiating charges XIII

The electron has an accelerated motion thus emitting the so-called **cyclotron radiation** at the cyclotron frequency ω_C , the same frequency of the rotating electron. Cyclotron radiation is elliptically polarized. When observing in the plane perpendicular to the magnetic field, the polarization is linear and is directed along the same plane. On the contrary, when observing along the direction of the magnetic field vector, the polarization is circular, clockwise as seen from one side and counterclockwise from the other. In term of Stokes parameters, one has

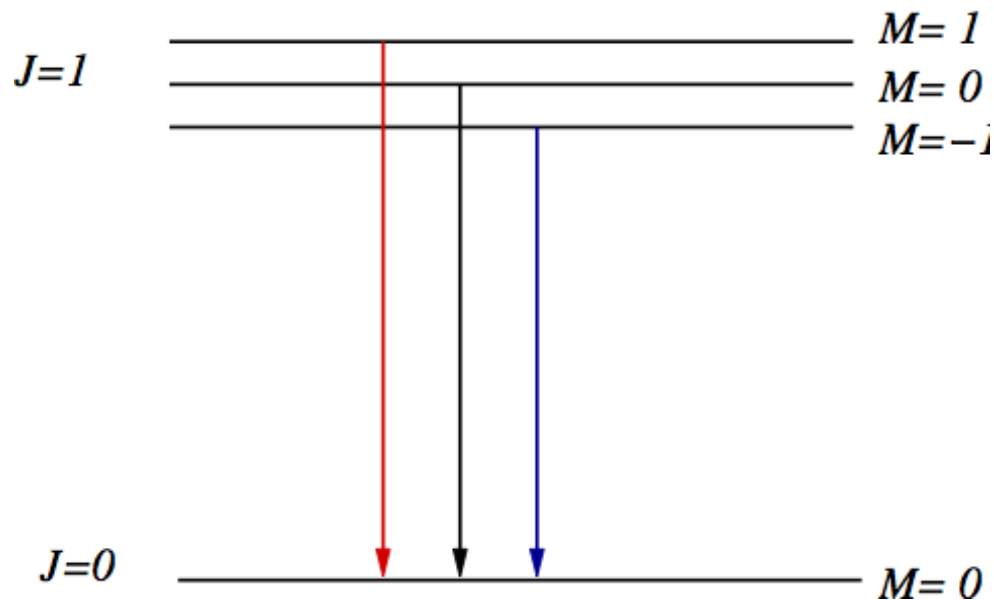
$$\begin{pmatrix} I_\omega \\ Q_\omega \\ U_\omega \\ V_\omega \end{pmatrix} = \frac{1}{8\pi} \frac{e_0^2 v_\perp^2 \omega_C^2}{c^3 R^2} \delta(\omega - \omega_C) \begin{pmatrix} 1 + \cos^2 \theta \\ -\sin^2 \theta \\ 0 \\ -2 \cos \theta \end{pmatrix}$$

Polarization in spectral lines I

In the spectropolarimetric analysis of laboratory plasmas and astrophysical objects, it is commonly observed that spectral lines are polarized, the polarization signature being, in general, different from line to line and variable along the line profile. Line polarization may be accompanied, or not, by continuum polarization. As typical examples of astrophysical plasmas, we can just mention sunspots and solar active regions, the higher layers of the solar atmosphere (including the chromosphere and the corona), magnetic stars, stars with extended envelopes, broad line regions of active galactic nuclei, astrophysical masers, etc. The physical mechanisms involved are different from object to object, but today it is possible to treat the general phenomenon of generation and transfer of polarized radiation within a unique theoretical framework.

Polarization in spectral lines II

For describing the way an atom can produce polarized radiation it is necessary to resort to a quantum description. The simplest atomic model that can be considered is the one of a two-level atom having angular momentum $J=0$ in its lower level and angular momentum $J=1$ in its upper level. The upper level is degenerate and is indeed composed of three sublevels, characterized by a further quantum number M that can assume the values 1, 0, and -1.

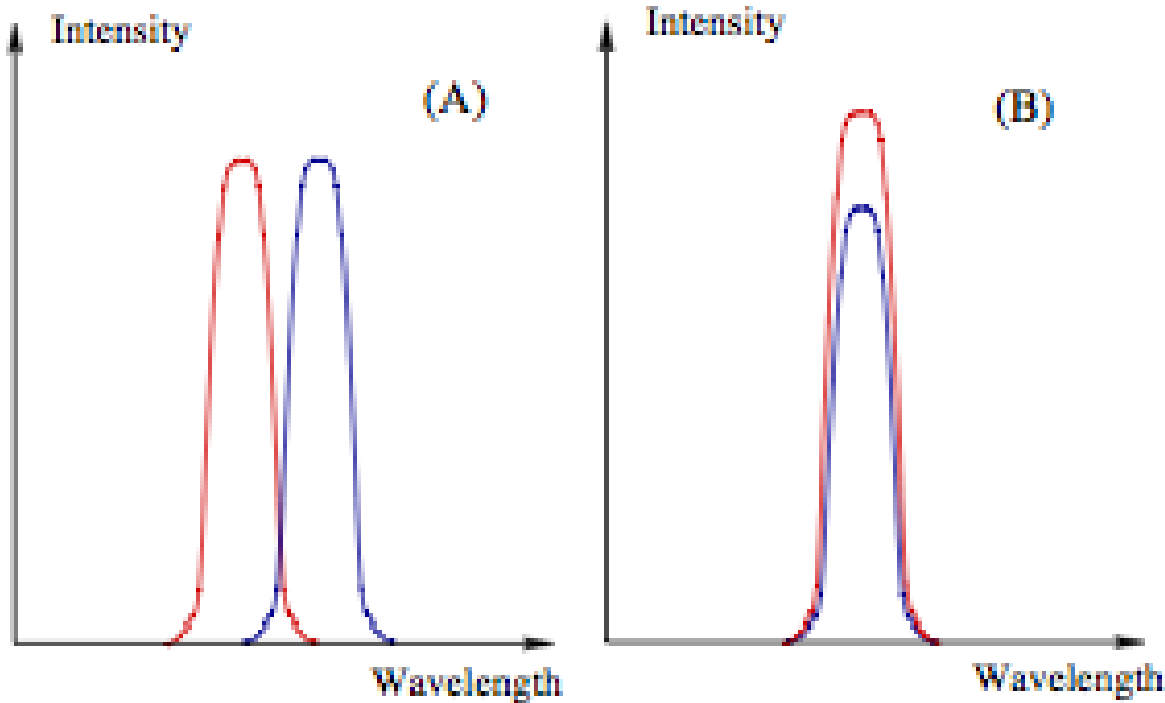


Polarization in spectral lines III

The three transitions correspond to different values of ΔM and each of them is characterized by a particular antenna diagram and a particular polarization of the emitted radiation. Transitions having $\Delta M=0$ behave, in emission as linear antennas, while transitions having $\Delta M=1$ or $\Delta M=-1$ behave as circular antennas of different handedness. For instance, looking along the direction of the quantization axis, the $\Delta M=0$ transition does not contribute to the emission (the antenna is seen pole-on) whereas the other two transitions emit radiation that is circularly polarized (positive circular polarization for the $\Delta M=1$ transition and negative circular polarization for the $\Delta M=-1$ transition.) If the levels are not split by a magnetic field (or by another physical agent) and if the magnetic sublevels are evenly populated the polarization washes out. Otherwise...

Polarization in spectral lines IV

- A) The levels are split but evenly populated
- B) The levels are degenerate but unevenly populated



In case A) there is circular polarization of one sign on a wing of the line and circular polarization of the opposite sign in the other wing. In the second case there is a net amount of polarization across the whole line.

Polarization in spectral lines V

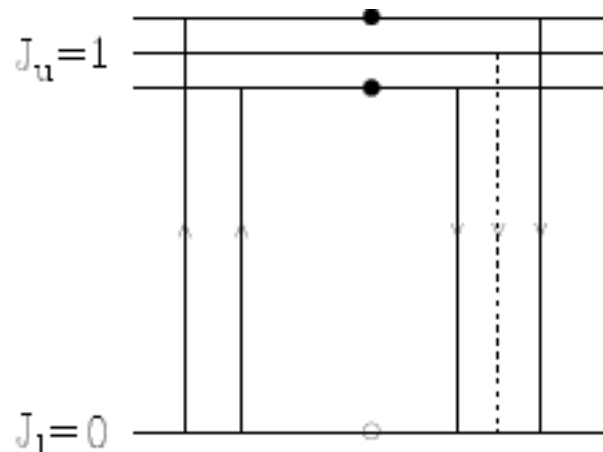
In case A) the polarization is due to splitting. This is the “classical” case of the Zeeman effect. In case B), on the contrary, we have a different phenomenon that is called

Atomic Polarization

The simplest manifestation of atomic polarization is the one that we have just seen in the former example. The degenerate sublevels of a given J-level can be unevenly populated and this results in the emission of polarized radiation. But how can this phenomenon be generated? How can an atom result in having its degenerate sublevels unevenly populated? A possibility is the one of irradiating it with anisotropic radiation. This possibility is referred to, in laboratory physics, with the name of **optical pumping**.

Polarization in spectral lines VI

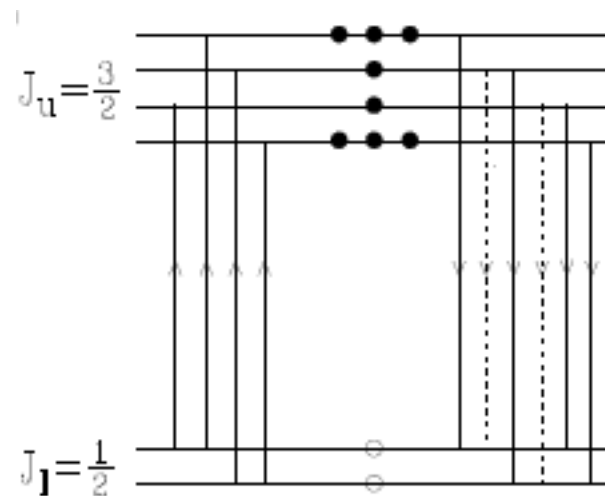
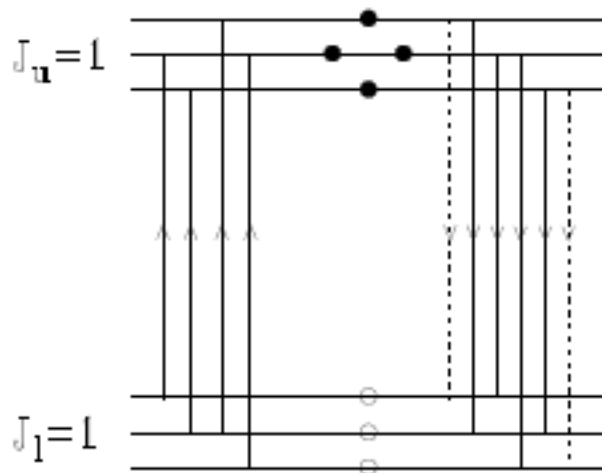
If the atom is illuminated by an unpolarized, unidirectional radiation beam, then, assuming the quantization direction along the incident beam, only transitions having either $\Delta M=1$ or $\Delta M=-1$ are efficient in populating the upper levels. This results in a considerable amount of atomic polarization, as here exemplified in the case of a 0-1 transition.



Polarization in spectral lines VII

For more "exotic" transitions the situation comes out to be more complicated....

To treat the phenomena of atomic polarization it is necessary to resort to a deeper description of the excitation of an atom which goes beyond the simple one of the "unpolarized case" where only the populations of the levels are specified. In general, one has to specify the populations of the single sublevels and, also, their phase relationships (coherence).



Polarization in spectral lines VIII

The most suitable theoretical tool to deal with these phenomena of atomic polarization is the density-matrix operator, defined in quantum mechanics by the equation

$$\rho = \sum_{\alpha} p_{\alpha} |\psi^{(\alpha)}\rangle \langle \psi^{(\alpha)}|$$

where p_{α} is the probability of finding the atomic system in the pure state α and where $|\psi^{(\alpha)}\rangle$ is the corresponding wave-function in Dirac's notations. The diagonal matrix elements of the density-matrix operator $\langle JM|\rho|JM\rangle$ represent the populations of the single sublevels, whereas the non-diagonal elements $\langle JM|\rho|JM'\rangle$ with $M \neq M'$ represent the coherence.

Polarization in spectral lines IX

The density-matrix operator was introduced in the astrophysical literature by M. Litvak in connection with his studies of polarization in astrophysical masers. The first application in solar physics is found in the thesis of Véronique Bommier's, devoted to the study of polarization in the D3 line of helium in prominences. Since then it has become a common tool for all the scientists working on the subject of resonance scattering in the solar atmosphere, with particular emphasis on the so-called second solar spectrum (the spectro-polarimetric spectrum of the Sun observed close to the limb).



Polarization in spectral lines X

The phenomena connected with atomic polarization are very important in the physics of resonant scattering. This is also because atomic polarization is sensitive to the presence of a magnetic field. This physical phenomenon was discovered by the German physicist W. Hanle around 1920, and now bears his name. In modern terms, we can state that the Hanle effect consists in the relaxation of atomic coherence due to the presence of a magnetic field.



Polarization in spectral lines X

Due to the Hanle effect, when a scattering process takes place in a magnetic environment, the scattered polarization results in being modified from its "magnetic-field-free" value. This allows to use the Hanle effect as a diagnostic tool for measuring magnetic fields.

You will surely hear much more about the Hanle effect in this Symposium. If you want to know more....

