AY 122: HOMEWORK 3
6 FEBRUARY 2009, COB

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At the Owens Valley Radio Observatory (OVRO[1]) we have a number of telescopes. The biggest is the 40-m telescope. This telescope is designed to work up to a frequency of 30 GHz. Estimate the following:

a The primary beam size at 10 GHz. [5]
b The gain of the telescope (K/Jy). [5]
c The 3-σ point source (Jy) sensitivity assuming a system temperature of 40 K, a bandwidth of 300 MHz and integration time of 10 s. [5]

2. Beam Efficiency: A Toy but hopefully Illustrative Problem.

An idea that has been around and something that I occasionally think about is an off-axis telescope located in the Antartic, the Himalayan range or the Andean range. Such a telescope could be optimized for $10\,\mu$m.

Consider a 1-dimensional telescope of length 10-m and the operating wavelength is say 1 cm. Assume that the surface is super smooth in so far as the observing wavelength of $10\,\mu$m is concerned. Furthermore, let us assume that the prime focus is off-axis and thus we will not have to deal with blockage by feed legs etc. As you well know the point spread response, $P(\theta)$ is the Fourier transform of the illumination function. The intensity response is given by $I_P = P(\theta)P^*(\theta)$.

a. Plot $I_P(\theta)$ assuming that the primary surface is uniformly illuminated. Estimate the main beam efficiency[2]. [5]

Date: 30 January 2009.

[1]This is the same facility that you saw in the movie *The Arrival*. In order to avoid legal problems the facility was renamed as ORO. The Aliens end up destroying the 40-m telescope with their universal micro-black-hole centrifugal vacuum cleaner.

[2]For consistency let us define Ω_M, the main beam solid angle as the integral of $I_P(\theta)/I(0)$ between angles at which the power response is (say) 50% of the peak value. The full beam solid angle, Ω_A is the integral of $I(\theta)/I(0)$ over all θ. The main beam efficiency is defined as the ratio of Ω_M to Ω_A. The integral for Ω_A converges rather slowly. It is acceptable that the integration limits go to say five times the main beam full width at half maximum.

b. Now assume that a hole of radius 1-m has been cut (to allow for Cassegrain operation). Plot $I_P(\theta)$ and estimate the main beam efficiency and the aperture efficiency[3]. Comment on your answer. [5]

c. Now we will inject a bit more reality. Assume that the feed at the prime-focus illuminates the primary surface with a Gaussian profile,

$$e(x) = \exp(-x^2/2\sigma^2).$$

Here x is the coordinate along the primary surface and the primary surface lies between $x = -D/2$ m and $x = +D/2$ m; here D is the diameter of the telescope and is set to 10 m. The units of σ are meters. $e(x)$ peaks at the center of the primary surface and its full-width at half maximum is 2.35σ. Furthermore, we will assume that the aperture has no Cassegrain hole. The goal is to estimate main-beam efficiency and aperture efficiency for a variety of assumed σ. To estimate the aperture efficiency you need to properly normalize the power response. First assume the illumination provided by the feed onto the primary is uniform with a constant= 1. Let the resulting power response be $I'(\theta)$. Now compute the power response assuming illumination $e(x)$ for σ ranging from 2-m to 8-m in steps of 1-m. Let the resulting power response be $I_G(\theta)$. We will use the metric $I_G(0)/I'(0)$ as a surrogate for aperture efficiency.

[15]

Note: You will note that the aperture efficiency is low for small σ and for large σ. In the case of small σ you are not using much of the primary collecting area. In the case of large σ much of the feed power is being "spilled" on to the ground. The really bad thing about spillover is the resulting increase in the system noise.

3. Basic Interferometry

The VLA consists of 27 antennas each of diameter 25-m. It operates over a wide range in frequency, from 300 MHz to 20 GHz. The bandwidth is limited to a maximum of 50 MHz. It has four configurations, A, B, C, D with the maximum radius of 30 km, 10 km, 3 km, and 1 km.

a. Estimate the primary beam size, the resolution and the delay beam at 1.4 GHz for the four configurations. [10]

b. What is the point source sensitivity of the VLA for 10 minutes of observation. Assume T_{sys} is 40 K. This is a reasonable value for the frequency range 1-10 GHz. [5]

c. What is the surface brightness sensitivity (per resolution beam) for a 12-hr observation (frequency=1.4 GHz and array=A, B, C, D)? [5]

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[3] Define the aperture efficiency as the ratio of the peak of $I(\theta)$ to the peak obtained by assuming full and uniform illumination.

4. Who said it is all relative?

Determing the anisotropy in the Cosmic Background Radiation (CBR) is one of the principal focus of cm-wave astronomy. The most well known isotropy is the dipole anisotropy caused by our motion of about 300 km s^{-1} with respect to the frame of the CBR. Assume that you have a receiver operating at 30 GHz, bandwidth 1 GHz, $T_{sys} = 30$, attached to a 5-m telescope. These parameters are similar to the kind of system that Readhead's group operates at OVRO.

a. Assuming you do only two measurements (assume that the direction of motion is known and thus the two measurements will be directed towards the two poles in the dipole direction – in the direction with the highest brightness temperature and in the opposite direction), estimate the amount of time you would need to make a 30-σ detection of our inferred motion.
b. You repeat the experiment 6 months later. Do you have enough sensitivity to see Earth's motion? Explain in words and justify with some calculations.
c. Detection of anisotropy on a 1-degree scale is important to understand Galaxy formation. So you decide to map a small patch of sky – 12 beam positions, separated by one beamdwidth and with each beam position observed for the same length of time, T. What is the value of T so that you are sensitive to variations, $\Delta T_{\rm CBR}/T_{\rm CBR} \sim 10^{-6}$ per beam?
d. Discuss some causes why the above calculations are too simplistic and there are other significant sources of error (other than radio metric noise). See Readhead et al. ApJ 346, pp566.

5. Mars

At cm-wavelengths, Mars can be considered to be a black body with an effective temperature of 216 K. Mars is usually used as a flux density calibrator at CARMA millimeter wave interferometer which consists of six 10-m and nine 6.1-m antennas. systems with typical parameters, $T_{\rm rec} = 200$ K, continuum bandwidth of 500 MHz, aperture effeciency of 0.5 and beam effeciency of 0.6, for frequencies around 100 GHz.

a For a 10-m telescope, estimate the system temperature, assuming a typical opacity of 0.1 due to the atmosphere and a 10-K contribution from spillover.
b What is the flux density of Mars at conjunction (i.e. Mars and the Sun at the same geocentric longitude) and opposition?
c Estimate the antenna temperature due to Mars on April 1, 2008.