

3.6.4 : Time-Dependent Equations for Multiple Ion States

In many cases, the two state solution is insufficient to estimate the shape of the recombination delay function. For higher Z elements, such as Silicon, more than two ions may contribute significantly to the distribution of fractional abundances at any given value for U (see figure 3-3). In particular, for $\log(U) \sim -1.5$, the ions Si^{+3} , Si^{+4} , and Si^{+5} each make up $\gtrsim 20\%$ of all silicon atoms. Si IV is a common and easily measured BAL, so, with this as our motivation, we attempt a solution for the time-dependent photoionization/recombination equations for more than two ion states.

Adopting a notation similar to that used in §3.6.2, for element X of atomic number Z , the ions $X^{+0}, X^{+1}, \dots, X^{+Z}$, have fractional abundances x_0, x_1, \dots, x_Z , respectively. Note that we cannot use a dimensionless time τ in the same way as in §3.6.2 since each ion has a different characteristic timescale for recombination. Otherwise, the same conventions as in §3.6.2 are used here, $\alpha_R(i)$ is the recombination rate for ion i , and γ_i and Γ_i^* depend on the incident flux and the photoionization cross section for ion i . We further define $R_i \equiv n_e \alpha_R(i)$, and $P_i \equiv \gamma(i) \Gamma_i^*$, for recombinations of ion $i+1$ to i , and photoionizations of ion i to $i+1$, respectively. Γ_i^* are constants which will be determined by the equilibrium condition at $t = 0$.

For the middle ions (not neutral or fully ionized), we have four terms contributing to the change in x_i at any instant of time: (1) an increase due to recombinations of $i+1$ to i , (2) a decrease due to photoionizations of i to $i+1$, (3) an increase due to photoionizations of $i-1$ to i , and (4) a decrease due to recombinations of i to $i-1$.

The set of Z coupled, first-order, ordinary differential equations, and the condition of conservation of ions yields: